*Materials Transactions*, Vol. 54, No. 3 (2013) pp. 269 to 275


# Influence of Copper Volume Fraction on Tensile Strain/Stress Tolerances of Critical Current in a Copper-Plated DyBCO-Coated Conductor

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The influence of the volume fraction ($V_f$) of copper, plated at room temperature over a $DyBa_2Cu_3O_{7-\delta}$-coated conductor, on the tensile strain tolerance and stress tolerance of critical current at 77 K was studied over a wide range of copper $V_f$ values. The copper plating exerts a tensile stress during cooling because copper has a higher coefficient of thermal expansion than the substrate conductor. Before application of tensile strain, the copper plated at room temperature yielded at 77 K when the copper $V_f$ was lower than a critical value, and was in an elastic state at 77 K when the copper $V_f$ was higher than the critical value. The strain tolerance of critical current increased with increasing copper $V_f$ due to an increase in thermally induced compressive strain in the substrate tape. The stress tolerance of critical current decreased with increasing copper $V_f$ because copper is softer than the substrate tape. These results, together with the trade-off between strain tolerance and stress tolerance (i.e., stress tolerance decreases with increasing strain tolerance), were analyzed by modeling. The results show that the restriction imposed by the trade-off, which limits the ability to simultaneously obtain a high strain tolerance and a high stress tolerance, can be relaxed by strengthening the copper. [doi:10.2320/matertrans.MBW201201]

(Received September 24, 2012; Accepted November 16, 2012; Published December 28, 2012)

***Keywords:*** *coated conductor, copper plating, critical current, strain tolerance, stress tolerance*

## 1. Introduction

The critical current of RE(Y, Sm, Dy, Gd)$Ba_2Cu_3O_{7-\delta}$-coated conductors under externally applied strain first drops at the point of irreversible strain at which the superconducting layer cracks, and then drops further with increasing strain due to crack extension.[1–11] For safety and reliability, a high irreversible strain and hence a high strain tolerance are required. One way to raise the strain tolerance is to impart a compressive strain to the superconducting layer in the current transport direction. It has been shown that for YBCO-coated conductor, an addition of a copper layer enhances the irreversible strain limit.[1] This enhancement stems from a thermally induced compressive strain in the YBCO layer that, because copper has a higher coefficient of thermal expansion than substrate tape, occurs during cooling to cryogenic temperature.[1] However, the stress-bearing capacity of copper-added composite tape lowers with increasing copper volume fraction because copper is softer than the substrate tape. Thus, it is necessary to elucidate the influence of copper volume fraction on not only the strain tolerance of the critical current, but also its stress tolerance.

Recently, we attempted (1) to estimate experimentally the change in strain/stress tolerances of critical current with the volume fraction of a copper layer electroplated onto a $DyBa_2Cu_3O_{7-\delta}$ (DyBCO) substrate tape and (2) to describe the trade-off between the strain tolerance and stress tolerance (strain tolerance increases with increasing copper volume fraction, but stress tolerance decreases) by modeling.[2] We found that at 77 K, the copper continues to impart a compressive strain to the substrate tape and hence to the superconducting layer,[1] and that tolerant stress decreases with increasing copper volume fraction. These experimental results can be satisfactorily explained by the modeling analysis.

However, in the preceding work,[2] the volume fraction of copper was limited to two values (0.427 and 0.546) and data could only be obtained in cases where copper yielded during cooling to 77 K. The present work was conducted with the following aims: (1) to collect additional experimental data by widely varying the volume fraction of copper (as to include cases in which copper remains in an elastic state after cooling to 77 K); and (2) by mechanical modeling for a wide range of copper volume fractions, to analyze experimental results of the change in tolerant strain and tolerant stress with increasing volume fraction and the trade-off between tolerant strain and tolerant stress.

## 2. Experimental Procedure

$DyBa_2Cu_3O_{7-\delta}$ (DyBCO)/Hastelloy C-276-coated conductor, prepared at THEVA, Germany,[12] was used as a substrate tape in this work, as in our preceding work.[2] The substrate tape consisted of a Hastelloy C-276 substrate (thickness 90 µm), a MgO buffer layer (3.3 µm) deposited by inclined substrate deposition, a MgO cap layer (0.3 µm), a DyBCO superconducting layer (2.5 µm) and a silver layer (0.5 µm). The critical current of the substrate tape at zero applied strain was ~210 A.

Copper was electroplated onto the substrate tape at room temperature with an electrolyte composed of $CuSO_4{\cdot}5H_2O$: 225 g/l, $H_2SO_4$: 35 ml/l and distilled water. This and other preparations were done at Kyoto University. In order to uniformly plate copper onto the substrate tape, a pure copper

*Present: Professor Emeritus, Kyoto University plate with an original thickness 0.2 mm was wound to round tubular shape and, to serve as an anode, placed into the bath so as to surround the substrate tape. In the preceding work,[2] we measured the stress–strain curve and critical current for three sets of specimens of different plated copper volume fractions ($V_{Cu} = 0$, 0.427 and 0.546). In the present work, two additional sets of specimens ($V_{Cu} = 0.301$ and 0.648) were prepared and tested.

Uniaxial tensile strain was applied to the specimen at 77 K with an Instron-type testing machine. The specimen was gripped by copper chucks, which also served as current electrodes during critical current measurement. To reduce stress concentrations within and near the grips, indium metal foils were inserted between the specimen and the grips. Specimen strain was measured with a couple of very light Nyilas-type extensometers.[13] Stress was monitored with a load cell at each applied strain. Voltage taps for the measurement of voltage ($V$)–current ($I$) curves were soldered with a spacing of 15 mm along the gage length of the extensometers. $V$–$I$ curves under various applied tensile strains were measured by the usual four-probe method at 77 K in a self-magnetic field. Critical current $I_c$ was estimated under a criterion of 1 μV/cm from the measured $V$–$I$ curve at each applied strain. Stress ($\sigma_C$) and critical current ($I_c$) were measured simultaneously at each tested strain ($\varepsilon_C$). All results obtained in the present work and also our preceding[2] work, which together cover the range $V_{Cu} = 0$ to 0.648, were subject to analysis in this work.

It is noted that when substrate tape without a copper plating is tested under an applied tensile strain, quenching occurs after a several-percent reduction in critical current, due to the progression of multiple cracks near the gripped portion and also due to the small thickness (0.5 μm) of the silver layer.[5,6] With copper plating, the variation in critical current with applied tensile strain was successfully measured over a wide range of applied strains and critical currents.[2–4] Copper plating acts not only to improve irreversible strain characteristics but also to provide a shunting circuit for imposed current, which helps to suppress quenching.[3,4]

## 3. Results and Discussion

### 3.1 Stress–strain range where cracking of DyBCO layer initiates and extends

An example of the measured change in critical current $I_c$ and tensile stress $\sigma_C$ of the copper-plated conductor (hereinafter referred to as composite tape) at 77 K with increasing applied tensile strain $\varepsilon_C$ is shown in Fig. 1. The open circle shows the stress–strain range in which cracking of the superconducting layer initiates and propagates, causing a reduction in critical current. The following features are found in this example.

(1) The stress–strain relation was almost straight up to around $\varepsilon_C = 0.4\%$, at which point Hastelloy C-276 began to macroscopically yield.

(2) The measured critical current decreased sharply in the straight stress–strain region in advance of macroscopic yielding.

(3) The slope $S$ in the straight stress–strain region was read to be 154 GPa. As the composite tape was composed of substrate tape and plated copper, a two-component (substrate tape and copper plating) model could conceivably be used to analyze the stress–strain relation of the composite tape. If copper deforms elastically, the Young's modulus of the composite tape, $E_{C,I}$, can be expressed by

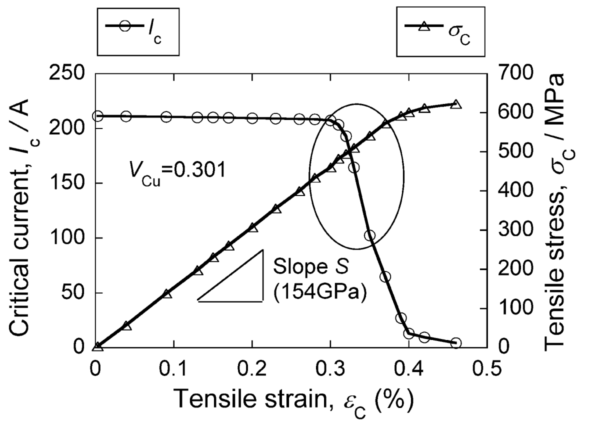


Fig. 1 An example of measured change in critical current $I_c$ and tensile stress $\sigma_C$ with increasing applied tensile strain $\varepsilon_C$. The open circle shows the stress–strain range in which cracking of the superconducting layer initiates and extends, causing a reduction in critical current.

$$E_{C,I} = E_{ST}(1 - V_{Cu}) + E_{Cu}V_{Cu}. \tag{1}$$

Here, $E_{ST}$ is the Young's modulus of substrate tape (= 216 GPa)[2] and $E_{Cu}$ is the Young's modulus of copper (= 117 GPa).[14] Substituting these values and $V_{Cu} = 0.301$ (the copper volume fraction of this specimen) into eq. (1), we arrive at $E_{C,I} = 186$ GPa. This value is excessively high relative to our measured value of $S = 154$ GPa. If the copper deforms plastically and if strain hardening is low in comparison with Young's modulus, the Young's modulus $E_{C,II}$ of the composite tape can be approximately expressed by

$$E_{C,II} = E_{ST}(1 - V_{Cu}). \tag{2}$$

Substituting the values mentioned above, we arrive at $E_{C,II} = 151$ GPa, which is close to the measured value of $S = 154$ GPa. This suggests that copper in this specimen was already in a yielded state at $\varepsilon_C = 0\%$; that is, before the application of tensile strain. In this way, we can judge whether the copper yielded at $\varepsilon_C = 0\%$ from the slope of the initial portion of the stress–strain curve.

### 3.2 Stress state of copper judged from the stress–strain relation

Figure 2 shows the relation between measured tensile stress ($\sigma_C$) and strain ($\varepsilon_C$) in early stages of deformation (up to $\varepsilon_C = 0.25\%$) at 77 K for five specimens: $V_{Cu}$ = (a) 0, 0.301, 0.427 and 0.546, and (b) 0.648. In the specimens with $V_{Cu} = 0$, 0.301, 0.427 and 0.546, stress $\sigma_C$ increased almost linearly with strain $\varepsilon_C$ (Fig. 2(a)). In the specimen with $V_{Cu} = 0.648$, the stress–strain relation was not linear (Fig. 2(b)). The difference in shape of these two sets of stress–strain relations indicates that the stress state of copper at $\varepsilon_C = 0\%$ is dependent on $V_{Cu}$.

Whether copper had yielded at $\varepsilon_C = 0\%$ was judged from slope ($S$) values as follows. Figure 3 shows the slope ($S$)

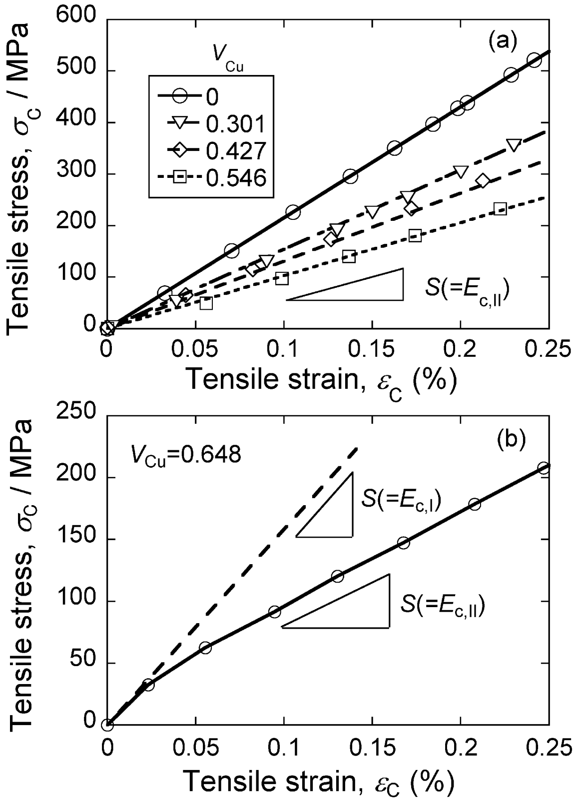


Fig. 2 Tensile stress $\sigma_C$-strain $\varepsilon_C$ relation in the early stage up to $\varepsilon_C = 0.25\%$ measured at 77 K for five specimens with $V_{Cu}$ = (a) 0, 0.301, 0.427 and 0.546; and (b) 0.648. The results for the specimens with $V_{Cu}$ = 0, 0.427 and 0.546 were taken from a preceding work.[2)]

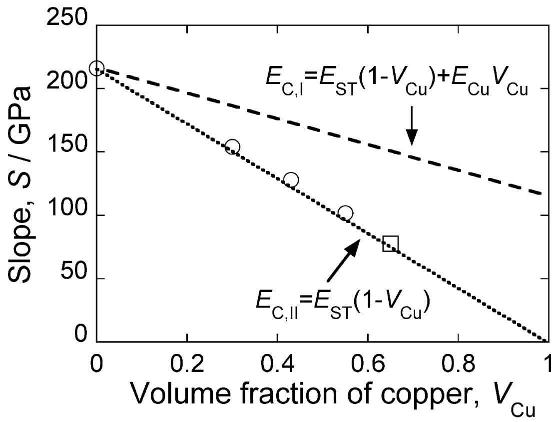


Fig. 3 Slope ($S$) values (open circles) of the specimens with $V_{Cu} = 0$, 0.301, 0.427 and 0.546 measured from the stress–strain curves in Fig. 2(a). The secondary modulus value $E_{C,II}$ of the sample with $V_{Cu} = 0.648$ in Fig. 2(b) is also shown with an open rectangle for reference. The $E_{C,I}$-$V_{Cu}$ and $E_{C,II}$-$V_{Cu}$ relations were calculated by the rule of mixtures (eqs. (1) and (2)) and are presented for comparison.

values (open circles) of the $V_{Cu} = 0$, 0.301, 0.427 and 0.546 specimens. The $E_{C,I}$-$V_{Cu}$ and $E_{C,II}$-$V_{Cu}$ relations calculated by eqs. (1) and (2), respectively, are also shown for comparison. The slope values (open circles) of these specimens accord well to $E_{C,II}$. This result clearly shows that copper had yielded in tension at $\varepsilon_C = 0\%$. In other words, the residual stress of copper was almost equal to its yield stress, since the plated copper in this work was soft (as shown later) and its strain hardening was not dominant (Fig. 3). On the other hand, the specimen with $V_{Cu} = 0.648$ showed a higher slope at low strain ($\varepsilon_C <$ approximately 0.02%) and a lower slope at higher strains (Fig. 2(b)). The initial slope within the low strain range was almost the same as that calculated by eq. (1). This means that copper was in an elastic state at $\varepsilon_C = 0\%$ in the case of a high $V_{Cu}$ fraction (0.648) but, in the case of lower $V_{Cu}$ fractions (0.301, 0.427 and 0.546), had already yielded. The slope in the higher strain range ($\varepsilon_C > 0.02\%$) for the $V_{Cu} = 0.648$ specimen was read to be 77.6 GPa, which is represented by an open rectangle in Fig. 3. The measured slope fell on the calculated $E_{C,II}$-$V_{Cu}$ line, demonstrating that copper had yielded at approximately $\varepsilon_C = 0.02\%$. These results means that when $V_{Cu}$ is high, the thermally induced stress in the copper is low, the copper is in an elastic state, and the copper is not in a yielded state at $\varepsilon_C = 0\%$.

### 3.3 Dependence of stress state of copper at 77 K on copper volume fraction

In the present work, copper was electro-plated at room temperature onto the substrate tape. The resulting assembly (i.e., the composite tape) was then cooled to 77 K for measurement of critical current. The copper plate imparts a compressive strain on the substrate tape during cooling, since the coefficient of thermal expansion ($\alpha$) of copper ($\alpha_{Cu} = 16.8 \times 10^{-6}$/K[15)]) is higher than that of the substrate tape, which mainly consists of Hastelloy C-276 ($\alpha_{ST} = 11.3 \times 10^{-6}$/K[16)]).

While the composite tape is being cooled from room temperature to 77 K, Hastelloy C deforms elastically over the whole temperature range due to its high yield stress (750 MPa in the present specimen[2)]). On the other hand, copper is rather soft and hence two cases can take place, as discussed in Subsection 3.2: case (A), in which the copper is in an elastic state, and case (B), in which the copper has yielded. With regard to case (A), thermally induced residual strains in the substrate and in the copper are hereinafter denoted as $\Delta\varepsilon_{ST,r,A}$ and $\Delta\varepsilon_{Cu,r,A}$; with regard to case (B), as $\Delta\varepsilon_{ST,r,B}$ and $\Delta\varepsilon_{Cu,r,B}$, respectively. We make this distinction because each case entails the use of a different equation to express residual strain.

In case (A), the coefficient of thermal expansion of the composite, $\alpha_C$, is expressed by

$$\alpha_C = \frac{\alpha_{ST}E_{ST}(1 - V_{Cu}) + \alpha_{Cu}E_{Cu}V_{Cu}}{E_{ST}(1 - V_{Cu}) + E_{Cu}V_{Cu}} \tag{3}$$

where $\alpha$, $E$ and $V$ refer to the coefficient of thermal expansion, Young's modulus and volume fraction, respectively, and the subscripts ST and Cu refer to the substrate tape and copper, respectively. Noting the temperature difference as $\Delta T$ ($-216$ K in this work; = 77 to 293 K (room temperature)) and using eq. (3) for $\alpha_C$, the residual strains at 77 K of copper ($\Delta\varepsilon_{Cu,r,A}$) and substrate tape ($\Delta\varepsilon_{ST,r,A}$) are expressed by

$$\begin{aligned}\Delta\varepsilon_{Cu,r,A} &= (\alpha_C - \alpha_{Cu})\Delta T \\ &= \frac{(\alpha_{ST} - \alpha_{Cu})E_{ST}\Delta T(1 - V_{Cu})}{E_{ST}(1 - V_{Cu}) + E_{Cu}V_{Cu}}\end{aligned} \tag{4}$$

$$\begin{aligned}\Delta\varepsilon_{ST,r,A} &= (\alpha_C - \alpha_{ST})\Delta T \\ &= \frac{(\alpha_{Cu} - \alpha_{ST})E_{Cu}\Delta T V_{Cu}}{E_{ST}(1 - V_{Cu}) + E_{Cu}V_{Cu}}.\end{aligned} \tag{5}$$

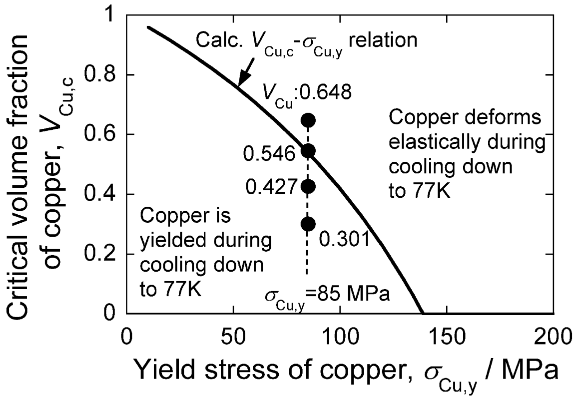


Fig. 4 Calculated critical volume fraction of copper $V_{Cu,c}$ as a function of copper yield stress $\sigma_{Cu,y}$. Above the $V_{Cu,c}$-$\sigma_{Cu,y}$ relation, copper is in an elastic state at 77 K in the as-cooled condition. Below the $V_{Cu,c}$-$\sigma_{Cu,y}$ relation, copper is in a plastic state at 77 K in the as-cooled condition. The closed circles show the $V_{Cu}$ values of the present specimens. They indicate that at 77 K and before an application of tensile strain, the copper in the $V_{Cu} = 0.648$ specimen is in an elastic state and the copper in the $V_{Cu} = 0.301$, 0.427 and 0.546 specimens is in a plastic state.

The thermally induced residual stress $\Delta\sigma_{Cu,r,A}$ in copper at 77 K is given by $E_{Cu}\Delta\varepsilon_{Cu,r,A}$. When $\Delta\sigma_{Cu,r,A}$ reaches the yield stress $\sigma_{Cu,y}$ of copper, the copper will yield. When the volume fraction of copper $V_{Cu}$ is high, $\Delta\sigma_{Cu,r,A}$ becomes low and hence the copper tends to be in an elastic state. Thus, when $V_{Cu}$ is high, case (A) appears. On the other hand, when $V_{Cu}$ is low, case (B) appears. The critical volume fraction of copper $V_{Cu,c}$, below which the copper yields (at 77 K and $\varepsilon_C = 0\%$) and above which the copper remains elastic (at 77 K and $\varepsilon_C = 0\%$), is calculated by substituting $\Delta\varepsilon_{Cu,r,A} = \sigma_{Cu,y}/E_{Cu}$ into eq. (4).

$$V_{Cu,c} = \frac{\sigma_{Cu,y}E_{ST} - (\alpha_{ST} - \alpha_{Cu})E_{ST}E_{Cu}\Delta T}{\sigma_{Cu,y}(E_{ST} - E_{Cu}) - (\alpha_{ST} - \alpha_{Cu})E_{ST}E_{Cu}\Delta T} \quad (6)$$

By substituting $E_{ST} = 216\,\text{GPa}$, $E_{Cu} = 117\,\text{GPa}$, $\alpha_{ST} = 11.3 \times 10^{-6}/\text{K}$, $\alpha_{Cu} = 16.8 \times 10^{-6}/\text{K}$ and $\Delta T = -216\,\text{K}$ into eq. (6), we calculate $V_{Cu,c}$ as a function of $\sigma_{Cu,y}$. The result is shown in Fig. 4. $V_{Cu,c}$ decreases with increasing $\sigma_{Cu,y}$ and reaches zero at $\sigma_{Cu,y} = 139\,\text{MPa}$. This phenomena stems from the fact that the maximum thermally induced strain in copper is limited to $(\alpha_{ST} - \alpha_{Cu})\Delta T$, which is obtained by substituting $V_{Cu} \approx 0$ into eq. (4). As a result, when $\sigma_{Cu,y}$ is higher than the maximum exerted stress of copper ($= E_{Cu}(\alpha_{ST} - \alpha_{Cu})\Delta T = 139\,\text{MPa}$), copper behaves elastically during cooling and remains in an elastic state at $\varepsilon_C = 0\%$ at 77 K.

The calculated $V_{Cu,c}$-$\sigma_{Cu,y}$ relation is a boundary, below which the copper yields during cooling and above which the copper does not yield. In the present work, case (A) was observed to apply to the specimen with $V_{Cu} = 0.648$ and case (B) was observed to apply to the specimens with $V_{Cu} = 0.301$, 0.427 and 0.546, as shown in Subsection 3.2. As will be shown later in Subsection 3.5, the yield stress of copper $\sigma_{Cu,y}$ was estimated to be 85 MPa by an analysis of the dependence of tolerant strain on $V_{Cu}$. At $\sigma_{Cu,y} = 85\,\text{MPa}$ in Fig. 4, volume fraction $V_{Cu} = 0.648$ is beyond the boundary and accordingly case (A) takes place. Likewise, volume fractions $V_{Cu} = 0.301$, 0.427 and 0.546 are below or on the boundary and accordingly case (B) takes place. In this way we can account for the difference in stress state judged from the stress–strain curve in Subsection 3.2.

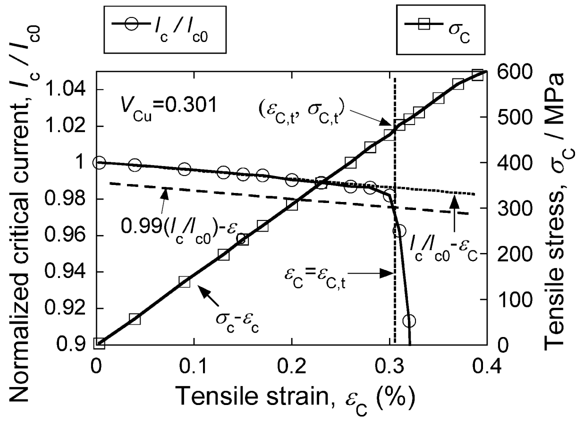


Fig. 5 Graphical representation for determining the tolerant strain $\varepsilon_{C,t}$ and tolerant stress $\sigma_{C,t}$ of critical current from measured $I_c/I_{c0}$-$\varepsilon_C$ and $\sigma_C$-$\varepsilon_C$ relations.

### 3.4 Measurement of tolerant strain and tolerant stress

The strain dependence of $I_c$ in the reversible strain range has been investigated for wide variety of RE (Y, Sm, Dy, Gd) BCO samples.[1–11] Reported results show that the shape of the $I_c$-$\varepsilon_C$ curve within the reversible strain range is dependent on the species of RE (Y, Dy, Sm, Gd), the fabrication process and microstructure, residual strain, and so on. Concerning the present DyBCO substrate tape without plated copper, it has been shown that $I_c$ decreases almost linearly[2–6] within the reversible strain range. However, it is not known whether the $I_c$-$\varepsilon_C$ relation is strictly linear or slightly curved. This point makes it difficult to determine the irreversible limit. In the preceding work,[2] we defined tolerant strain in a simplified manner. In the present work, we use the same approach. This procedure is briefly outlined as follows.

Figure 5 shows an example of the measured change in (1) normalized critical current $I_c/I_{c0}$ ($I_{c0}$: critical current at $\varepsilon_C = 0\%$) and (2) stress $\sigma_C$ as a function of strain $\varepsilon_C$. It provides a graphical means of determining tolerant strain $\varepsilon_{C,t}$ and tolerant stress $\sigma_{C,t}$ from measured $I_c/I_{c0}$-$\varepsilon_C$ and $\sigma_C$-$\varepsilon_C$ relations. The measured $I_c/I_{c0}$ decreases almost linearly with increasing $\varepsilon_C$ (as shown with a dotted line labeled $I_c/I_{c0}$-$\varepsilon_C$) and then drops sharply. Taking a 99% offset line (broken line labeled $0.99(I_c/I_{c0})$-$\varepsilon_C$) of the measured $I_c/I_{c0}$-$\varepsilon_C$ relation in the reversible strain range, we can identify the tolerant strain ($\varepsilon_{C,t}$) value as the intersection of the measured $I_c/I_{c0}$-$\varepsilon_C$ curve and the 99% offset line. In advance of the application of this criterion, we checked the reversibility of critical current by measuring critical current under loading and partial unloading. By this we confirmed that the tolerant strain obtained by the 99% offset criterion is surely just beyond the irreversible strain.[2] We utilize this 99% $I_c/I_{c0}$ offset line criterion in this work as well.

Figure 6 shows the change in normalized critical current $I_c/I_{c0}$ with tensile strain $\varepsilon_C$ in the specimens with $V_{Cu} =$ (a) 0, (b) 0.301, (c) 0.417, (d) 0.546 and (e) 0.648. Tolerant strain $\varepsilon_{C,t}$ obtained by this 99% $I_c/I_{c0}$ offset line criterion is indicated with an arrow for each specimen. Once the $\varepsilon_{C,t}$

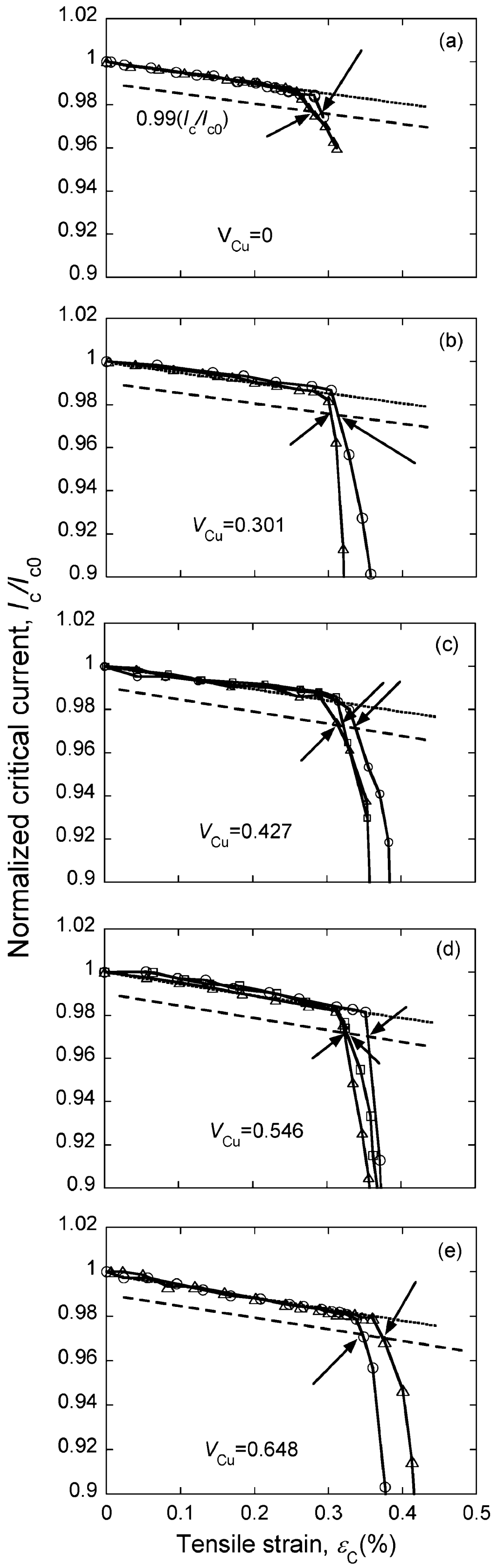


Fig. 6 Normalized critical current $I_c/I_{c0}$-tensile strain $\varepsilon_C$ relation as measured for specimens with $V_{Cu}$ = (a) 0, (b) 0.301, (c) 0.427, (d) 0.546 and (e) 0.648. For each specimen, the strain tolerance $\varepsilon_{C,t}$ of critical current is indicated with an arrow.

value is obtained, the tolerant stress $\sigma_{C,t}$ at the corresponding $\varepsilon_{C,t}$ value can be read from the stress–strain relation of Fig. 5.

Figure 7 shows measured tolerant strain $\varepsilon_{C,t}$ and tolerant stress $\sigma_{C,t}$, plotted against copper volume fraction ($V_{Cu}$). This

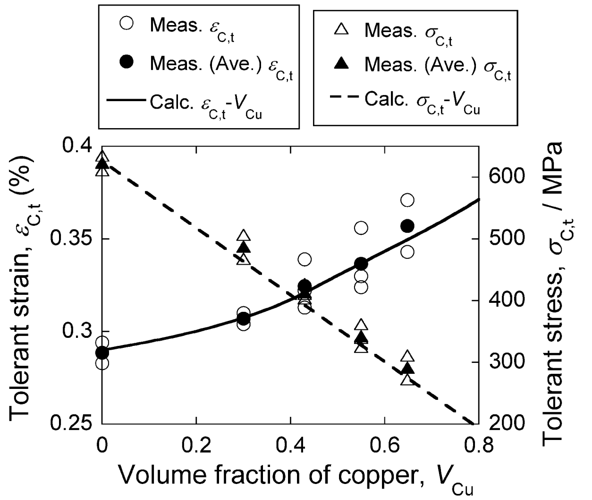


Fig. 7 Estimated values of tolerant strain $\varepsilon_{C,t}$ and tolerant stress $\sigma_{C,t}$, plotted against copper volume fraction ($V_{Cu}$). The solid curve and broken line refer to the analyzed $\varepsilon_{C,t}$-$V_{Cu}$ and $\sigma_{C,t}$-$V_{Cu}$ relations, respectively.

relation will be used in Subsection 3.5 to estimate the yield stress of copper $\sigma_{Cu,y}$. Then, in Subsection 3.6, we will use this estimated $\sigma_{Cu,y}$ value to quantitatively analyze the experimentally obtained $\varepsilon_{C,t}$-$V_{Cu}$ and $\sigma_{C,t}$-$V_{Cu}$ relations from a mechanical viewpoint. The trade-off between $\varepsilon_{C,t}$ and $\sigma_{C,t}$ ($\varepsilon_{C,t}$ increases but $\sigma_{C,t}$ decreases with increasing $V_{Cu}$) will be also discussed in Subsection 3.6.

### 3.5 Estimation of yield stress of copper by the obtained tolerant strain data in case (B)

As shown in Subsections 3.2 and 3.3, the copper had already yielded before application of tensile strain at 77 K in the composite specimens with $V_{Cu} = 0.301$, 0.427 and 0.546. Namely, case (B) took place. In this subsection, the $\varepsilon_{C,t}$-$V_{Cu}$ relation of these specimens is used to estimate the yield stress of copper. Under the applied tensile strain, the copper and substrate tape in these specimens deform plastically and elastically, respectively. By noting the residual strain of the substrate tape in the composite ($\Delta\varepsilon_{ST,r,B}$), we can express the stress of composite $\sigma_C$ as a function of $\varepsilon_C$ in the form:

$$\sigma_C = E_{ST}(\varepsilon_C + \Delta\varepsilon_{ST,r,B})(1 - V_{Cu}) + \sigma_{Cu,y}V_{Cu}. \quad (7)$$

The terms $E_{ST}(\varepsilon_C + \Delta\varepsilon_{ST,r,B})$ and $\sigma_{Cu,y}$ in eq. (7) indicate the stresses of the substrate tape and copper in the composite tape, respectively, both at an applied strain $\varepsilon_C$. As $\sigma_C = \varepsilon_C = 0$ before the test begins, eq. (7) can be reduced to $E_{ST}(1 - V_{Cu})\Delta\varepsilon_{ST,r,B} + \sigma_{Cu,y}V_{Cu} = 0$, from which $\Delta\varepsilon_{ST,r,B}$ for yielded copper is expressible as

$$\Delta\varepsilon_{ST,r,B} = -\left(\frac{\sigma_{Cu,y}}{E_{ST}}\right)\left(\frac{V_{Cu}}{1 - V_{Cu}}\right). \quad (8)$$

The tolerant strain $\varepsilon_{C,t}$ of the composite tape with plated copper is given by $\varepsilon_{C,t} = \varepsilon_{ST,t} - \Delta\varepsilon_{ST,r,B}$, where $\varepsilon_{ST,t}$ is the tolerant strain of the substrate tape alone. Substituting $\Delta\varepsilon_{ST,r,B}$ given by eq. (8) into this relation, we have

$$\varepsilon_{C,t} = \varepsilon_{ST,t} + \left(\frac{\sigma_{Cu,y}}{E_{ST}}\right)\left(\frac{V_{Cu}}{1 - V_{Cu}}\right). \quad (9)$$

Equation (9) is valid in case (B) only. The value of $\varepsilon_{ST,t}$, which corresponds to the value of $\varepsilon_{C,t}$ at $V_{Cu} = 0$ in Fig. 7,

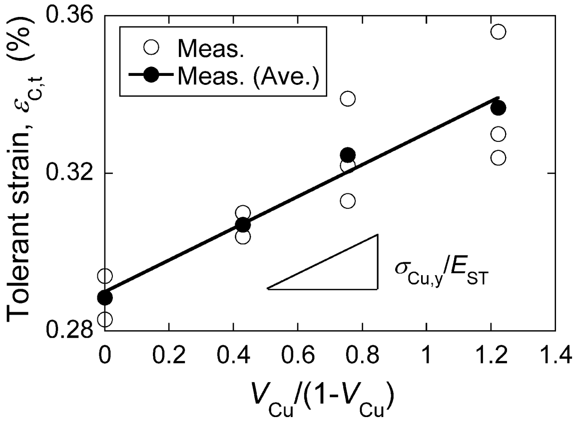


Fig. 8 Plot of the measured tolerant strain $\varepsilon_{C,t}$ against $V_{Cu}/(1-V_{Cu})$ for estimation of copper yield stress $\sigma_{Cu,y}$. The slope corresponds to $\sigma_{Cu,y}/E_{ST}$ (eq. (9)).

was found to be 0.289% on an average. Figure 8 plots measured $\varepsilon_{C,t}$ values (Fig. 7) against $V_{Cu}/(1-V_{Cu})$. The slope corresponds to $\sigma_{Cu,y}/E_{ST}$ (eq. (9)). As $E_{ST}$ is known to be 216 GPa, the value of $\sigma_{Cu,y}$ can be obtained from the slope value. The $\sigma_{Cu,y}$ value obtained by this method with the present specimens was 85 MPa.

### 3.6 Description of experimentally obtained $\varepsilon_{C,t}$-$V_{Cu}$ and $\sigma_{C,t}$-$V_{Cu}$ relations and the trade-off between $\varepsilon_{C,t}$ and $\sigma_{C,t}$

In case (A), where copper has not yielded under thermally induced stress, residual strain $\Delta\varepsilon_{ST,r,A}$ in the substrate is given by eq. (5). As the tolerant strain $\varepsilon_{C,t}$ of the composite tape is given by $\varepsilon_{C,t} = \varepsilon_{ST,t} - \Delta\varepsilon_{ST,r,A}$, we can express $\varepsilon_{C,t}$ as follows:

$$\varepsilon_{C,t} = \varepsilon_{ST,t} - \frac{(\alpha_{Cu} - \alpha_{ST})E_{Cu}\Delta T V_{Cu}}{E_{ST}(1-V_{Cu}) + E_{Cu}V_{Cu}}. \qquad (10)$$

Equation (10) is valid in case (A) only. The $\varepsilon_{C,t}$ for case (B) has been given by eq. (9). In case (A), copper deforms elastically in, at least, the early stage of deformation under an applied tensile stress. At a high applied stress, the copper yields when the sum of the residual stress and tensile stress exerted on the copper reaches the yield stress of copper ($\sigma_{Cu,y} = 85$ MPa). In the elastic deformation stage, the stress ($\sigma_C$)–strain ($\varepsilon_C$) relation is given by

$$\begin{aligned}\sigma_C &= E_{ST}(\varepsilon_C + \Delta\varepsilon_{ST,r,A})(1-V_{Cu}) + E_{Cu}(\varepsilon_C + \Delta\varepsilon_{Cu,r,A})V_{Cu} \\ &= \{E_{ST}(1-V_{Cu}) + E_{Cu}V_{Cu}\}\varepsilon_C. \qquad (11)\end{aligned}$$

Here, $E_{ST}\Delta\varepsilon_{ST,r,A}(1-V_{Cu}) + E_{Cu}\Delta\varepsilon_{Cu,r,A}V_{Cu}$ is held at zero, since the residual stresses of substrate tape and copper are balanced. In eq. (11), the stress $E_{Cu}(\varepsilon_C + \Delta\varepsilon_{Cu,r,A})$ of elastically deforming copper increases with increasing $\varepsilon_C$. When $\varepsilon_C + \Delta\varepsilon_{Cu,r,A}$ reaches $\varepsilon_{Cu,y}$ (yield strain of copper), copper yields. $\varepsilon_C$ at copper yielding, $\varepsilon_{C,Cu,y}$, is given by $\varepsilon_{C,Cu,y} = \varepsilon_{Cu,y} - \Delta\varepsilon_{Cu,r,A}$. The stresses of the substrate ($\sigma_{ST,Cu,y}$) and composite ($\sigma_{C,Cu,y}$) at copper yielding are expressed by

$$\sigma_{ST,Cu,y} = E_{ST}(\varepsilon_{Cu,y} - \Delta\varepsilon_{Cu,r,A} + \Delta\varepsilon_{ST,r,A}) \qquad (12)$$

$$\begin{aligned}\sigma_{C,Cu,y} &= E_{ST}(\varepsilon_{Cu,y} - \Delta\varepsilon_{Cu,r,A} + \Delta\varepsilon_{ST,r,A})(1-V_{Cu}) \\ &\quad + \sigma_{Cu,y}V_{Cu}. \qquad (13)\end{aligned}$$

When $\varepsilon_C$ exceeds $\varepsilon_{C,Cu,y}$, copper deforms plastically. The stress of the composite where copper deforms plastically is given by

$$\begin{aligned}\sigma_C &= \sigma_{C,Cu,y} + E_{ST}\{\varepsilon_C - (\varepsilon_{Cu,y} - \Delta\varepsilon_{Cu,r,A})\}(1-V_{Cu}) \\ &= E_{ST}(\varepsilon_C + \Delta\varepsilon_{ST,r,A})(1-V_{Cu}) + \sigma_{Cu,y}V_{Cu}. \qquad (14)\end{aligned}$$

In all specimens within the present and preceding works, the tolerant stress of the composite was in the range of copper plastic deformation. The stress $\sigma_C$ of the composite tape reaches the tolerant stress $\sigma_{C,t}$ when $\varepsilon_C$ reaches $\varepsilon_{C,t}$. Substituting $\sigma_C = \sigma_{C,t}$, $\varepsilon_C = \varepsilon_{C,t} = \varepsilon_{ST,t} - \Delta\varepsilon_{ST,r,A}$ into eq. (14) in case (A) and substituting $\sigma_C = \sigma_{C,t}$, $\varepsilon_C = \varepsilon_{C,t} = \varepsilon_{ST,t} - \Delta\varepsilon_{ST,r,B}$ into eq. (7) in case (B), we arrive at eq. (15) for the tolerant stress $\sigma_{C,t}$ common to cases (A) and (B):

$$\sigma_{C,t} = E_{ST}\varepsilon_{ST,t}(1-V_{Cu}) + \sigma_{Cu,y}V_{Cu}. \qquad (15)$$

Equation (15) means that (a) tolerant stress $\sigma_{C,t}$ is independent of residual strain when copper has yielded either during cooling or under tensile strain, while tolerant strain $\varepsilon_{C,t}$ ($= \varepsilon_{ST,t} - \Delta\varepsilon_{ST,r,A}$ in case (A) and $\varepsilon_{ST,t} - \Delta\varepsilon_{ST,r,B}$ in case (B)) is dependent on residual strain; and (b) $\sigma_{C,t}$ decreases linearly with increasing $V_{Cu}$.

When the yield stress of copper $\sigma_{Cu,y}$ is known (85 MPa for the present specimens), the critical volume fraction $V_{Cu,c}$ of copper can be estimated with eq. (6). Cases (A) and (B) arise in the $V_{Cu}$ range of $V_{Cu} > V_{Cu,c}$ and $V_{Cu} < V_{Cu,c}$, respectively. Tolerant strain $\varepsilon_{C,t}$ is thus calculated by eq. (10) in case (A) and by eq. (9) in case (B). Also, the $\varepsilon_{C,t}$-$V_{Cu}$ relation is calculated by taking $V_{Cu}$ as a variable for a given $\sigma_{Cu,y}$ value. The $\sigma_{C,t}$-$V_{Cu}$ relation can be calculated as a function of $V_{Cu}$ by eq. (15) for both cases (A and B). The following values mentioned above were used for calculation of $\varepsilon_{C,t}$-$V_{Cu}$ and $\sigma_{C,t}$-$V_{Cu}$ relations: $\varepsilon_{ST,t} = 0.289\%$, $E_{ST} = 216$ GPa, $E_{Cu} = 117$ GPa, $\alpha_{ST} = 11.3 \times 10^{-6}$/K, $\alpha_{Cu} = 16.8 \times 10^{-6}$/K, $\Delta T = -216$ K and $\sigma_{Cu,y} = 85$ MPa. The calculated $\varepsilon_{C,t}$-$V_{Cu}$ and $\sigma_{C,t}$-$V_{Cu}$ relations are shown with solid and broken lines, respectively, in Fig. 7. They satisfactorily describe our experimental results. That is, the present approach makes it possible to describe the change in $\varepsilon_{C,t}$ and $\sigma_{C,t}$ with $V_{Cu}$ for wide range of $V_{Cu}$ covering both cases (A) and (B).

The $\sigma_{C,t}$-$\varepsilon_{C,t}$ relation was obtained from the calculated $\varepsilon_{C,t}$-$V_{Cu}$ and $\sigma_{C,t}$-$V_{Cu}$ relations in Fig. 7. The $\sigma_{C,t}$-$\varepsilon_{C,t}$ relation thus obtained, together with experimental results (closed circles), is shown with a broken line in Fig. 9. The experimental results are satisfactorily described. Within the present specimens, the yield stress of copper $\sigma_{Cu,y}$ was not high (85 MPa). Accordingly, $\sigma_{C,t}$ decreased sharply with increasing $\varepsilon_{C,t}$. The relation between $\sigma_{C,t}$ and $\varepsilon_{C,t}$ is trade-off ($\sigma_{C,t}$ decreases with increasing $\varepsilon_{C,t}$). This trade-off limits our ability to simultaneously achieve high values both for $\varepsilon_{C,t}$ and $\sigma_{C,t}$. One way to relax this restriction is to raise the yield stress of copper. For comparison, a calculated $\sigma_{C,t}$-$\varepsilon_{C,t}$ relation for an assumed value of $\sigma_{Cu,y} = 300$ MPa is shown as a solid curve in Fig. 9. Under this condition of $\sigma_{Cu,y} = 300$ MPa, copper is in an elastic state at $\varepsilon_C = 0\%$ and accordingly $\varepsilon_{C,t}$ is given by eq. (10) for any volume fraction of copper $V_{Cu}$. However, copper yields under tensile strain and accordingly, at ($\varepsilon_C$, $\sigma_C$) = ($\varepsilon_{C,t}$, $\sigma_{C,t}$), copper enters a plastic state and hence eq. (15) becomes applicable for the calculation of $\sigma_{C,t}$ values. The calculated $\sigma_{C,t}$-$\varepsilon_{C,t}$ curve for $\sigma_{Cu,y} = 300$ MPa is higher than that for $\sigma_{Cu,y} = 85$ MPa, indicating that the replacement

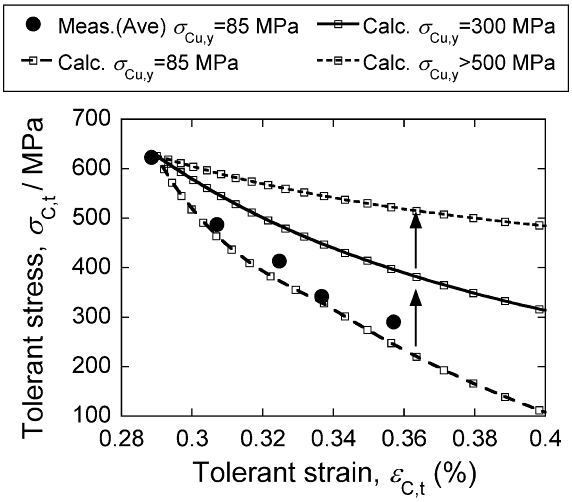


Fig. 9 Measured trade-off between tolerant stress $\sigma_{C,t}$ and tolerant strain $\varepsilon_{C,t}$ (closed circles) where $\sigma_{Cu,y}$ was 85 MPa. Also shown are calculated relations for $\sigma_{Cu,y} = 85$ MPa, $\sigma_{Cu,y} = 300$ MPa and $\sigma_{Cu,y} > 500$ MPa.

of a low yield stress copper by a high yield stress copper is effective in relaxing this trade-off.

When $\sigma_{Cu,y}$ is even higher, for instance, when $\sigma_{Cu,y}$ is above 500 MPa, copper behaves elastically at $(\varepsilon_C, \sigma_C) = (\varepsilon_{C,t}, \sigma_{C,t})$. In such a case, $\varepsilon_{C,t}$ is given by eq. (10). Substituting eq. (10) for $\varepsilon_C = \varepsilon_{C,t}$ into eq. (11), we have

$$\sigma_{C,t} = \{E_{ST}(1 - V_{Cu}) + E_{Cu}V_{Cu}\} \times \left[\varepsilon_{ST,t} - \frac{(\alpha_{Cu} - \alpha_{ST})E_{Cu}\Delta T V_{Cu}}{E_{ST}(1 - V_{Cu}) + E_{Cu}V_{Cu}}\right]. \quad (16)$$

From the $\varepsilon_{C,t}$-$V_{Cu}$ relation calculated by eq. (10) and the $\sigma_{C,t}$-$V_{Cu}$ relation calculated by eq. (16), we determined a $\sigma_{C,t}$-$\varepsilon_{C,t}$ relation for $\sigma_{Cu,y} > 500$ MPa (dotted curve in Fig. 9). As both the substrate and the copper behave elastically, the $\sigma_{C,t}$-$\varepsilon_{C,t}$ relation is not dependent on $\sigma_{Cu,y}$ value and accordingly is the same for any $\sigma_{Cu,y}$ value greater than 500 MPa. The $\sigma_{C,t}$-$\varepsilon_{C,t}$ curve shifts upward when $\sigma_{Cu,y}$ is raised from 85 to 300 MPa and shifts further upward when $\sigma_{Cu,y}$ is further raised (>500 MPa), as shown with the arrows in Fig. 9. In this way, the use of a stronger copper is an effective way to relax the restrictions of the trade-off relation.

## 4. Conclusions

(1) Stress–strain relations and stress/strain tolerances of critical current of copper-plated DyBCO-coated conductor tape were obtained over a wide range of copper volume fractions.

(2) Analysis of the measured stress–strain relation at 77 K and calculation of thermally induced stress during cooling from room temperature to 77 K revealed that (a) when the volume fraction of copper is lower than a critical value, the copper, which had been plated at room temperature, has already yielded at 77 K before any application of tensile strain and (b) when the volume fraction of copper is higher than a critical value, the copper, which, as above, had been plated at room temperature, remains elastic at 77 K.

(3) The strain tolerance of critical current increases with increasing copper volume fraction because of an increase in thermally induced compressive strain in the substrate tape. This experimental result was satisfactorily described by a mechanical modeling approach that incorporates the thermally induced stress–strain behavior of the substrate tape and copper over a wide range of copper volume fractions.

(4) The stress tolerance of critical current decreased with increasing copper volume fraction because copper is softer than substrate tape. The decrease in stress tolerance with an increase in copper volume fraction was almost linear. The measured dependence of stress tolerance on copper volume fraction was satisfactorily described by the present modeling approach.

(5) The trade-off between the strain tolerance and stress tolerance (stress tolerance decreases with increasing strain tolerance) was clearly detected in the present experiments over a wide range of copper volume fractions. The measured trade-off relation was satisfactorily described by the modeling analysis. Our modeling analysis also showed that the restriction of this trade-off, which limits our ability to obtain a high strain tolerance and high stress tolerance together, can be relaxed by using stronger copper.


## Acknowledgements

The authors thank the Ministry of Education, Culture, Sports, Science and Technology, Japan, for a Grant-in-Aid (no. 22360281).


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