Appropriateness of the Hencky Equivalent Strain as the Quantity to Represent the Degree of Severe Plastic Deformation

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The Hencky strain is a logarithmic strain extended to a three-dimensional analysis. Although Onaka has shown that the Hencky equivalent strain is an appropriate measure of large simple-shear deformation (2010), Jonas et al. (2011) have recently presented a paper claiming that the application of the Hencky strain to large simple-shear deformation is in error. In the present paper, it is shown that the claim of Jonas et al. is contrary to recent accepted knowledge on the Hencky strain. [doi:10.2320/matertrans.M2012077]

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1. Introduction

Logarithmic strain or true strain is an appropriate measure to describe large deformations of materials. The logarithmic strain, treated as a scalar, is widely used to describe the one-dimensional extension of a rod. Hencky, in 1928, extended the strain to three-dimensional analysis by defining components for three principal axes.1

Onaka has calculated the Hencky strain tensor for large simple-shear deformation and shown that the Hencky equivalent strain is an appropriate measure of the deformation.2,3 However, Jonas et al.4 have recently presented a paper claiming that this application of the Hencky strain2,3) is in error. In the present paper, it is shown that the claim of Jonas et al.4) is not consistent with recent accepted knowledge on the Hencky strain. I hope that advantages of the Hencky strain and its equivalent strain can now be appreciated properly.

2. The Hencky Strain and Its Equivalent Strain

The deformation-gradient tensor \( F \) is a fundamental quantity for defining strain. \( F \) is decomposed via the polar decomposition theorem5) into:
\[
F = RU = VR,
\]
where \( R \) is an orthogonal tensor and the right-stretch tensor \( U \) and the left-stretch tensor \( V \) are symmetric tensors with positive eigenvalues. The tensor \( R \) represents pure rotation in \( F \). On the other hand, \( U \) and \( V \) represent the essential deformation in \( F \). We can exclude the effects of pure rotation in \( F \) if we define the strain using \( U \) or \( V \). The Lagrangean Hencky strain \( h_L \) and the Eulerian Hencky strain \( h_E \) are defined as6)
\[
h_L = \ln U \quad \text{and} \quad h_E = \ln V,
\]
where \( \ln U \) and \( \ln V \) are the logarithms of the matrices \( U \) and \( V \).

Figure 1 shows the operation of \( F \) in the case of simple-shear deformation. The operations of \( V \) giving \( h_E \) and \( R \) are also indicated in Fig. 1. Grids and ellipses around the grids in Fig. 1 show the shapes before and after the occurrence of simple-shear deformation. The axes \( x_i^E \) show the principal axes of \( h_E \) and the angle \( \alpha \) shows the direction of the \( x_2^E \) axis after the simple-shear deformation.

The equivalent strains of \( h_l \) and \( h_E \) are the same.2,3) The Hencky equivalent strain \( h_{eq} \) for simple-shear deformation has been calculated as:
\[
h_{eq} = \frac{2}{\sqrt{3}} \ln \left( \frac{\gamma}{2} + \sqrt{1 + \frac{\gamma^2}{4}} \right),
\]
where \( \gamma \) represents the amount of simple-shear deformation. The explicit expressions of \( F, R, U, V, h_l \) and \( h_E \) are shown in the Onaka papers.2,3)

To compare the degree of severe plastic deformation (SPD) produced by different processes, an appropriate scalar representing the degree of deformation is needed. Onaka has shown that \( h_{eq} \) is an appropriate quantity representing the degree of SPD.2,3) When deformation is large, it is well known that there are major problems with the nominal strain.2,3) The nominal equivalent strain \( \epsilon_{eq} \) for the simple-shear deformation is written as:
\[
\epsilon_{eq} = \frac{\gamma}{\sqrt{3}}.
\]
Since $h_{eq} \approx 2 \ln \gamma / \sqrt{3}$ when $\gamma \gg 1$ as shown by Ref. 2), the difference in $e_{eq}$ and $h_{eq}$ is substantial and $e_{eq}$ becomes much larger than $h_{eq}$. The nominal equivalent strain $e_{eq}$ is that Jonas et al.4) and their research group5) recommend to characterize large simple-shear deformation.

3. Rotation of the Principal Axes of Strain

In their paper,4) Jonas et al. have stated that the application of the Hencky formalism is not appropriate to describe simple-shear deformation because of the rotation of the principal axes of strain. This is related to conclusions (1) and (4) of their paper.4) The rotation of the principal axes of strain is the decrease in the angle $\alpha$ in Fig. 1 with increasing the amount of shear $\gamma$. As Jonas et al. have pointed out in their paper,5) it is true that the axes of the Hencky strain and its increment do not coincide during simple shear deformation. It is also true that, in papers published in the 1980s,6)(10,11) we find the consensus on the inapplicability of the Hencky strain to cases where the principal strain axes rotate during deformation. However, understanding on the Hencky strain has changed in the mid-1990s,12,13)

In a recent paper, Shrivastava et al.8) have also pointed out this limited applicability of the Hencky formulation. In a response to this paper, Onaka has shown14) that the Hencky strain is an appropriate measure of strain for large deformations including large simple-shear deformation. That is to say, the problems of the Hencky formulation pointed out by Jonas et al.5) and Shrivastava et al.9) on the strain increments have been already solved with the help of the $D$-frame12) or the log-spin13) concept. As shown in recent papers in the field of applied mechanics, for example in Ref. 15), the Hencky strain is considered to be a favored measure of strain because of its remarkable properties at large deformations. As well as the claim by Shrivastava et al.,9) the claim by Jonas et al.5) that the application of the Hencky strain to large simple-shear deformation is in error is contrary to recent accepted knowledge on the Hencky strain.14) When we treat large deformations, there are no rational reasons to use $e_{eq}$ instead of $h_{eq}$.

In the paper by Jonas et al.,4) they have not evaluated the degree of deformation, but have instead considered the equivalent strain increment and stress-strain relationship. This is evident from their conclusions (2) and (3).4) They have found that they cannot obtain reasonable stress–strain relationships from the equivalent strain increment of the Hencky strain.4) Here it is noted that unrealistic results are obtained by their method of analysis.

Figure 2 schematically shows the scope of application of various strains. The infinitesimal or nominal strain is used when we consider small strain and small rotation. Although the Almansi strain5) and Green strain5) are also for small strain, these can be used reasonably even if rotation is large. As shown by Fig. 2, the Hencky strain is a relevant measure to describe general deformations including large deformations where both strain and rotation are large.

4. Summary

Jonas et al. have recently (2011) presented a paper claiming that the application of the Hencky equivalent strain to large simple-shear deformation, which Onaka made in his recent papers (2010), is in error. However, the claim of Jonas et al. is contrary to recent accepted knowledge on the Hencky strain. As Onaka has previously shown, the Hencky equivalent strain is an appropriate quantity to represent the degree of severe plastic deformation based on large simple-shear deformation.

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