Water Drop Erosion on Turbine Blades: Numerical Framework and Applications

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When small droplets are formed in the wet steam stage of a steam turbine, they may impact the blade surface at a high velocity and repetitive impacts cause water drop erosion, which emerges as one of the primary reliability concerns of the turbine. We propose an effective numerical framework that couples fluid mechanics with solid mechanics. The movements of water drops in a blade channel are analyzed based on the solution of the flow field of water steam in turbine, and impact statistics such as impact frequency, velocity, and position are obtained as the working condition and particle size are varied. A nonlinear wave model is established for high velocity liquid-solid impact, from which the characteristic impact pressure in liquid and peak impact stress in solid are obtained; the solutions are then superimposed with the pathways of water particles, and a fatigue analysis is carried out to elucidate the mechanisms of water drop erosion. The lifetime map on a blade surface with two different materials (1Cr13 and Ti-6Al-4V) under typical working conditions are obtained, in terms of operation hours, and the most dangerous water drop erosion regions and operating conditions of the steam turbine are deduced.

Keywords: water drop erosion, liquid-solid impact, numerical simulation

1. Introduction

Near the exhaust of a steam turbine engine (also known as the penultimate or final stage), the low-pressure working condition favors phase transformation, and small water drops are produced by the condensation of steam. Once formed, the water drops move with the flow and some of them may impact the blade surface with a velocity over 200 m/s — upon consecutive impact on the turbine blades, the surface material may spall off and this is known as the water drop erosion which severely affects the system reliability (Fig. 1). According to experimental observations, the most favorable impact locations include the leading or trailing edges of back arc and inner arc, and the erosion rate is around 0.1 micron per hour.1) Therefore, water drop erosion emerges as one of the primary reliability concerns in a turbine and its mechanism must be sufficiently understood.

The water drop erosion is also a very complex problem that couples multiphase fluid flow with impact mechanics and fatigue analysis, which requires a seamless coupling between fluid and solid mechanics. First, it is important to solve for the trajectories and velocities of water drops, as well as the distribution of mass and size of the particles. While some water drops may follow the steam flow and exit the turbine, others may impact the blade surface and key information needs to be collected, including the impact frequency, location, velocity, which depend on the aforementioned variables. Finally, the liquid-solid impact problem imposes a tremendous challenge and an appropriate model is needed to solve for the distribution and magnitude of transient impact stresses in the solid. When the solution of the fundamental liquid-solid impact problem is superimposed with the impact statistics, an fatigue analysis may be carried out to evaluate the lifetime, most dangerous areas and working conditions. In this paper, we establish multidisciplinary computational and mathematical models toward such an objective. The flexible framework may also be extended to similar problems encountered in practice, such as rain drop erosion on civil structures or airplane engines.

2. Flow Field in a Wet Steam Turbine Stage

2.1 Model and computation method

For a long blade turbine stage, the Ma number varies from low at the inlet to high (~supersonic). The conventional approach2,3) cannot be directly applied to simulate the flow in a turbine stage that involves both static nozzles and rotating blades. In order to reduce computational cost, in this section, the compressible pressure correction method (SIMPLE) is combined with the mixing plane model1,4,5) the static and moving regions are treated separately and connected through the “mixing plane”. A series of techniques are proposed to take into account the various boundary con-
dictions that are related with different speed ranges in the nozzle and blade.

The traditional $K - \varepsilon$ turbulence model\(^\text{(2-6)}\) is employed in numerical study, which takes the general form of N-S equation:

$$
\frac{\partial}{\partial x} (\rho u \phi) + \frac{\partial}{\partial y} (\rho u v \phi) + \frac{\partial}{\partial z} (\rho u \phi)
= \frac{\partial}{\partial x} \left( \Gamma \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left( \Gamma \frac{\partial \phi}{\partial z} \right) + S_0 + S_1 \quad (1)
$$

where $u$, $v$, $w$ are the velocity components in $x$, $y$, and $z$ directions, respectively; $\rho$ is the density of fluid, $\Gamma$ is the generalized viscosity factor, $S_0$ is the source term caused by turbulence model; $S_1$ is the source term caused by other factors, and $\phi$ is the field variable, which can be a velocity component ($u$, $v$, $w$), temperature ($T$), turbulence energy ($K$), and turbulence energy dissipation ($\varepsilon$), etc., see Table 1. The first column is the field variable of the standard $K - \varepsilon$ turbulence model; the second column is the generalized viscosity factor; and the third column is the source term caused by turbulence model. In Table 1, $c_\mu = 0.09$, $c_1 = 1.44$, $c_2 = 1.92$, $c_\sigma = 1.0$, $c_\tau = 1.3$, $c_\rho = 1.0$. $Pr$ is the Prandtl number, $p$ is pressure, $\mu$ is the molecular viscosity factor, and $\mu_4$ is the turbulence viscosity factor.\(^{6)}\)

$$
\mu_t = c_\mu \rho K^2/\varepsilon. \quad (2)
$$

$$
\mu_{eff} = \mu + \mu_t \quad (3)
$$

$$
G = \mu_t \left\{ 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right\}. \quad (4)
$$

Since there are 8 variables in (1), $(\rho, p, u, v, w, K, \varepsilon, T)$, the system equation is needed to close the equation set. For wet steam, the second Virial coefficient equation is used:

$$
p = RT/\nu(1 + B/\nu), \quad (5)
$$

where $\nu$ is the kinematic-viscosity coefficient, and $R$ is the gas constant, and $B = 2.0624 \times 10^{-3} - \frac{2.0129}{T} \times 10^{\frac{6380}{T}}$.\(^ {7}\)

In this work, the coordinate system is fixed on the rotating blade. Consider a fluid block in a rotating system with angular velocity $\omega$, both the centrifugal force ($f_c$) and the Coriolis force ($f_e$) are acting on the block:

$$
f_e(x) = 0, f_c(y) = \omega^2(y - y_0), f_c(z) = \omega^2(z - z_0) \quad (6)
$$

$$
f_e(x) = 0, f_c(y) = 2\omega u, f_c(z) = -2\omega v \quad (7)
$$

where the direction of angular velocity vector is aligned with the $x$-axis, i.e. $\Omega = (\omega, 0, 0)$. The coordinates of the nozzle is $(y_0, z_0)$, thus, the polar radius vector is $r = (0, y - y_0, z - z_0)$. The source terms of the momentum equations become

$$
S_0 = 0, S_e = f_c(y) + f_c(y), S_w = f_c(z) + f_c(z). \quad (8)
$$

The inlet conditions include the temperature and velocity of the fluid. If the speed at the outlet approaches supersonic, the static inlet pressure must be given. The distribution of static pressure is given on the outlet, and the outlet velocity is assumed to be unidirectional;\(^{6)}\) other outlet variables are obtained by extrapolation. The fluid flow near the wall follows the wall function.\(^{6)}\) The static and dynamic regions are solved separately based on time-averaged parameters, and interacted through a mixing plane model.\(^{7)}\) In the physical space, the mixing plane (Fig. 1) is the narrow gap between the nozzle and blade, where the wet steam exits the nozzles and flows into the blade channels.

### 2.2 Flow field of wet steam flow in a turbine stage

The numerical framework established above is employed to simulate the flow field in a whole wet steam turbine stage, which is a penult stage of a 125 MW turbine (Dongfang Steam Turbine Company, China). The geometry and dimension are given in Fig. 2, where the height of nozzle, $l_1 = 450$ mm and the height of blade is $l_2 = 460$ mm. The outlet of nozzle channel and the inlet of blade channel are aligned with a plane in the radial direction, so as to apply the mixing plane model. Two operating conditions (Table 2) are simulated, under full designed and reduced (75%) loads, respectively. For simplicity, we consider only the flux of the working fluid (i.e. the inlet Ma number) is varied and all other parameters are fixed between these two working conditions. Note that the numerical protocol can be readily extended to other turbines with different dimensions and operating conditions.

The flow fields under the two operating conditions are shown in Fig. 3(a) and (b), respectively, for the top section of the stage. The color bar and arrow length denote the...
magnitude of velocity vectors. While the patterns of flow field in the nozzle channel are fairly close for the two working conditions, the flow fields in the blade channel are distinct. When the load of turbine varies, the flow speed at the inlet/outlet of the nozzle changes yet the angular velocity of blade is invariant. When these two factors are combined, both the magnitude and direction of the flow field in the blade channel become very sensitive to the load. Subsequently at reduced load, the inlet velocity vectors become opposite to the direction of rotation, which cause the inlet steam flow toward the back arc, and a backflow region is formed on the top section (Fig. 3(b)).

3. Motion of Water Drops in Blade Channel

3.1 Model and computation method

The diameter distribution of particles entering the blade channel is given by Shi. The particle trace model in the Lagrangian coordinates is employed in this study to obtain the statistics of movement of water drops in the blade channel. Since the typical humidity of wet steam is lower than 10% and the volume fraction of water drops in steam is under 0.01%, it is reasonable to assume that the motion of water drop would not counter-affect the fluid flow. We further neglect the crash and coalescence of water drops since the transient stress develops very quickly in the solid upon impact, and it is the most critical for erosion (see below).

We use a semi-random trace model which incorporates the correction of turbulence dissipation and with moderate computational cost. In the rotary coordinate system attached to the blade, the motion of particles is described as:

\[
\begin{align*}
\frac{du_p}{dt} &= \frac{1}{\tau}(u_g + u'_g - u_p)f(Re) \\
\frac{dv_p}{dt} &= \frac{1}{\tau}(v_g + v'_g - v_p)f(Re) + 2\omega w_p \\
\frac{dw_p}{dt} &= \frac{1}{\tau}(w_g + w'_g - w_p)f(Re) - 2\omega v_p + \omega^2 r_p
\end{align*}
\]

where \(\omega\) is the angular velocity of the coordinates (same as that of the blade), and \(r_p\) is the radial coordinate of the water drop, \(t = \rho_p d_p^2/(18\mu)\) is the relaxation time of particle

\[
f(Re) = \begin{cases} 
1 & Re < 1 \\
1 + \frac{Re^{2/3}}{6} & 1 \leq Re \leq 1000 \\
0.01854 Re & Re > 1000
\end{cases}
\]

(10)

where \(\rho_p\) is the particle density, \(d_p\) is the particle diameter, \(\mu\) is the dynamic-viscosity coefficient of gas. \(u_p, v_p, w_p\) are the velocity components of the particle, \(u_g, v_g, w_g\) are the velocity components of the fluid, and \(u'_g, v'_g, w'_g\) are the pulsant velocities of the turbulent flow. Assuming the turbulence is isotropic,

\[
\begin{align*}
\xi u' &= \frac{2}{3}K, & \xi v' &= \frac{2}{3}K, \\
\xi w' &= \frac{2}{3}K
\end{align*}
\]

(12)

where \(\xi\) is a random number with normal distribution, and it remains constant during a time step which tracks the movement of the particle. The turbulence energy \(K\) in the \(K - \epsilon\) turbulence model can be solved from the steam flow field (see Section 2). The velocities \((u_p, v_p, w_p)\) of the particle are obtained from (9); we then integrate the velocities \((u_p, v_p, w_p)\) to obtain the position coordinates \((x_p, y_p, z_p)\) of the particle.

3.2 Trace of water drops in blade channel

With the turbine stage and working conditions specified in Section 2.2, sample traces are shown in Figs. 4 and 5 at the bottom and top of blade channel, respectively. Combinations
of different particle size and working conditions are explored. Most 5 μm particles move along with the steam flow consistently, and only very few particles impact the inner arc of blade with small normal impact velocities. Because the inlet angles of flow are different, more 5 μm particles tend to impact the blade under the full load than that under reduced load (although the percentage of particles colliding with the blade is fairly small in both cases). By contrast, with the dominance of their inertia, the 100 μm particles can be easily separated from steam flow and impact the blade surface at a high normal velocity; in particular, under the reduced load, the head of back arc appears suffering to high-possibility of water drop impacts. We introduce a dimensionless impact rate \( \eta_n \):

\[
\eta_n = \lim_{\Delta A \to 0} \frac{\Delta N \cdot A_0}{\Delta A \cdot N_0}
\]  

(13)

where \( \Delta N \) is the number of drops collected by the element area, \( \Delta A \); \( N_0 \) is the total number of drops entering the channel, and \( A_0 \) is the area of the entry plane of the channel.

Figure 6 gives the maps of \( \eta_n \) of 100 μm on the inner and back arcs of the blade, and under both operating conditions. The x and z coordinates are normalized by the total length of blade in the x direction (x0) and height in the z direction (z0), respectively. The maximum impact rate with the reduced load is 2.6, which is greater than under full load. Even though the number of drops launched on the entry plane is same, the impact frequency on the head of back arc with operating condition No. 2 (Fig. 6(b)) is almost 5 times of that of No. 1 (Fig. 6(a)). In practice, the number of drops on the entry plane under the low load operating condition should be more than that of the full load condition, because the steam under the low load operating condition has lower enthalpy and more water drops will be produced. Thus, the head of back arc of blade is the most dangerous region that will be eroded by drops under the low load operating conditions. For the inner arc, the maximum impact frequency with full load is larger than that of reduced load. Under full load, the most critical erosion region is close to the diagonal line of the blade, whereas the region of concern upon reduced load is at the bottom of the blade.

Figures 7 and 8 give the maps of normal impact velocity distributions on the back arc/inner arc under the operating conditions No. 2, as the particle size is varied. In all cases, the impact velocity patterns of the larger particles are essentially the same when the diameter of drops is bigger than 50 μm. Since the larger particles are more critical for water drop erosion, once the diameter of drop exceeds 50 μm, the impact frequency and velocity distributions can be regarded as invariant so as to reduce the computational cost.

Therefore, for larger particles the distributions of impact velocity are independent of water drop diameters, and since they are also more critical, we mainly focus on the impact by larger particles in the following sections. At reduced load, the maximum impact velocity (Fig. 7) appears at the head of blade top, consistent with the highest impact frequency (Fig. 6(b)). Thus, this region is quite likely to subject to the most severe erosion; this agrees with.10) The maximum

Fig. 6 The distribution of \( \eta_n \) of 100 μm particles under operating condition (a) the back arc of blade of No. 1; (b) the back arc of blade of No. 2; (c) the inner arc of blade of No. 1; (d) the inner arc of blade of No. 2 (view in x-z plane).

Fig. 7 The distribution of normal impact velocity (m/s) on the back arc under the operating condition No. 2 (view in x-z plane), with the particle diameter: (a) 5 μm, (b) 10 μm, (c) 50 μm, (d) 150 μm, (e) 250 μm.

Fig. 8 The distribution of normal impact velocity (m/s) on the inner arc under the operating condition No. 2 (view in x-z plane), with the particle diameter: (a) 5 μm, (b) 10 μm, (c) 50 μm, (d) 150 μm, (e) 250 μm.
4. Liquid-Solid Impact

4.1 Fundamental considerations of modeling

There are very limited experimental studies on high velocity liquid-solid impact.\(^{11-13}\) Moreover, these experiments focused on the pressure field in the liquid, however the stress field in the solid is the most critical for causing water drop erosion. Since the surface material spalls off upon consecutive impact, the transient peak stress causing such damage due to impact must be below the surface. Therefore, effective theoretical model and numerical simulation are necessary for exploring the stress field and related erosion mechanisms. Due to the difficulty of coupling between liquid and solid phases, the study of liquid-solid impact is far less comparing with solid-solid impact counterparts.\(^{14,15}\)

Along previous analytical attempts, many researchers established simple models upon liquid-rigid surface interaction (many of them are 1D) and obtained the steady-state pressure.\(^{16-19}\) and then applied that pressure to obtain stress characteristics in an elastic half-space.\(^{16,17,20-23}\) Apparently, in these works, the pressure field in liquid was not simultaneously coupled with the stress wave in solid, the solutions were not valid for the realistic 3D condition, and the transient peak pressure in liquid and peak stress in solid, which are much more severe than the steady-state values,\(^{15}\) were not pursued.

In this section, we develop a fully-coupled, 3D dynamic solid-liquid model. On the interface between liquid and solid, all transient parameters, include pressure, force, mass point position and movement, are exchanged between the two regions and solved in situ. When a spherical water drop impacts on a solid plane, the shock wave forms inside the water drop due to the compressibility of liquid.\(^{24}\) The water drop thus imposes a pressure distribution on the surface of solid which varies with time and space, inducing stress waves which transmit through the solid, with possible formation and propagation of microcracks.

The Navier-Stokes (N-S) equation is the basic mathematical model for fluid:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0, \quad \frac{\partial \mathbf{V}}{\partial t} + \nabla \cdot (\mathbf{T} + p \mathbf{I} + \mathbf{D}) = 0 \quad (14)
\]

where \( \mathbf{V} \) is the velocity vector of mass point in liquid; the first equation of (14) is the continuity equation and the second equation is the momentum equation. Here, \( \mathbf{T} \) is the momentum tensor, \( p \mathbf{I} \) is the pressure tensor, and \( \mathbf{D} \) is the viscosity force tensor. \( \mathbf{I} \) is the Kronecker delta. Due to the complexity of the liquid-solid impact, a few basic assumptions are necessary for our model:

1. We assume normal impact is much more critical than tangential impact, and thus we focus only on the normal component of the impact velocity (since the tangential component does not produce impact stress). Thus we only focus on normal axisymmetric problem (Fig. 9) which simplifies formulation.

2. Due to the short characteristic impact duration (~ns) on turbine blade, the deformation in the metal is elastic for high-velocity impacts due to the strain rate effect.\(^{15}\) This greatly simplifies the current analysis and linear elasticity theory with small deformation can be adopted for the solid, and the coupling between the Euler coordinate system in liquid and Lagrange coordinate system in solid becomes easier.

3. Since the maximum pressure appears before the shock wave leaves the body of water drop,\(^{25,26}\) which will be validated in this paper, we do not consider subsequent events after the shock wave leaves the water drop. If the sonic speed \( c_E \) at the point \( e \) (Fig. 9, inside the edge cell \( E \)) is larger than the moving speed of the point \( (t_0) \), then the sonic wave pressure is faster than the edge of liquid, and the shock wave will break off the liquid edge, thus determining \( t_{0s} \), the critical moment when the shock wave breaks off.

4. From Section 3, at a high impact velocity up to 300 m/s, the contact angle \( \theta \) (Fig. 9) of the shock wave leaving the water drop is about \( 10^\circ \).\(^{20}\) Moreover, the water drop retains its spherical shape before the shock wave leaves;\(^{24}\) later we will also show that the contact angle is confined to a very small region before it leaves the water drop. Therefore, both the shape and volume variations of water sphere are negligible before the shock wave leaves the liquid drop,\(^{25,26}\) and thus the impact is an acoustic procedure with negligible viscosity effect. We may thus ignore the viscosity of liquid drop during high velocity impact (while retaining the compressibility of fluid), and reduce the N-S equation to the nonlinear wave equation.

4.2 3D Nonlinear wave model for liquid-solid impact

The coordinates in this problem are fixed at the interface of the undisturbed liquid, and thus we consider the problem where the solid is “impacting” the liquid (Fig. 9). The undisturbed fluid is static, \( V_0 = 0 \). The local density, pressure, and velocity in the liquid can be expressed as the summation of the values of undisturbed liquid (with subscript 0) and the relevant perturbation, which are all functions of time and space coordinates:

\[
\rho = \rho_0 + \Delta \rho, \quad p = p_0 + \Delta p, \quad \mathbf{V} = \mathbf{V}_0 + \Delta \mathbf{V} \quad (15)
\]

Substitute eq. (15) into eq. (14), note that \( \frac{\partial \rho_0}{\partial t} = 0, \quad \nabla \rho_0 = 0, \quad \nabla p_0 = 0, \quad \nabla \cdot \mathbf{T} \approx 0 \) after neglecting the higher order terms. After ignored the viscosity term \( \mathbf{D} \), eq. (14) becomes

![Fig. 9 The coordinates system of axisymmetric liquid-solid impact and coupling.](image-url)
\[ \frac{\partial (\Delta \rho)}{\partial t} + \nabla \cdot (\rho V) = 0 \]  
\[ \frac{\partial (\rho V)}{\partial t} + \nabla \cdot (\Delta \rho I) = 0 \]

(16a)  
(16b)

For a fluid, \( \Delta \rho \) can be written as 
\[ \Delta \rho = (\partial \rho / \partial \rho) \Delta \rho / c^2 \]

(17)

where \( c \) is the sonic speed in the fluid. Subtract the time derivative of eq. (16a) from the divergence of eq. (16b), and also subtract the time derivative of eq. (16b) from \( c^2 \) times the divergence of eq. (16a), and use eq. (17):

\[ \nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}, \quad \nabla^2 V = \frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} \]

(18)

These are the wave equations. Introduce the velocity potential function \( \Psi \) with \( V = \nabla \Psi \) and \( p = -\rho \cdot \partial \Psi / \partial t \), the wave equations of \( p \) and \( V \) are merged to one wave equation

\[ \nabla^2 \Psi = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2} \]

(19)

Since the present mathematical model of liquid-solid impact is based on the wave equation, it is also termed as “the wave model of liquid-solid impact”, or the “wave model”. Since the sonic speed \( c \) is a variable of the density (and thus pressure), the wave equation is a nonlinear second order PDE. According to the Tait state equation of water \( \rho = \rho(T) \), and the dimensionless coordinates \( X \) and \( R \), and the dimensionless stress \( \tau_x \) and \( \tau_r \):

\[ X = x/r_0, \quad R = r/r_0, \quad T = v_0 t/r_0 \]

(26)

where \( r_0 \) is the initial radius of liquid drop.

### 4.3 Simulations of water drop-1Cr 13/Ti-6Al-4V impact

Although the flexible model and numerical framework developed in this study can be readily extended to any liquid and elastic solid, we use two representative blade materials,
the high strength stainless steel 1Cr13 (ASTM #S41000) after quenching and tempering, and Ti-6Al-4V, to demonstrate the results of the liquid-solid impact. The main material properties (measured from experiments) are shown in Table 3, along with that of water.

As the particle size is varied, the dimensionless parameters, including the dimensionless time when shock wave breaks off the solid, maximum interface pressure and the time of its appearance, maximum stress and the time/position of breaks off the solid, maximum interface pressure and the time/position of its appearance, are found to be invariant and they are only functions of impact velocity. This finding is supported by experimental data in,\(^{20}\) where the water drops with different diameters have the same contact angle of lateral jetting. Thus, in the following examples, we only use one water drop size (1 mm) during simulation and all results are expressed in dimensionless terms.

We first fix the impact velocity at a representative \(v_0 = 100 \text{ m/s}\). Figure 10 shows a snap shot of the pressure distribution inside water drop and the stress field inside elastic solid, at 9 ns after impact. The pressure in water drop increases away from the origin, and within the contact area, the highest pressure is found at the edge. Semicircular-shaped pressure waves are formed in the disturbed liquid region because of the moving disturbing source (contact point), and the cap-like shock wave front is the envelope of the pressure wave. The stress distribution in solid exhibits an interference field of two symmetrical wave sources. There are two prominent high stress regions: the first one is near the contact edge, where the peak impact pressure is also found which acts as the primary wave source; the second one (which is important for erosion) is near the axis and below the surface, caused by the superposition of stress waves. For these two solid materials, the patterns of pressure and stress are slightly different, because their sonic speeds are relatively close. The stress in Ti-6Al-4V is a little bit lower than 1Cr13 because of its smaller modulus (see Table 3).

After the stress field of every time step is solved, the maximum equivalent stress point can be found and within each time step, this point should be regarded as the most dangerous point. If we plot the trace of such critical dangerous point for every time step, then the influence range of the critical impact stress can be known. Figure 11 shows the moving traces of the maximum equivalent stress point at each time step, with an interval \(\Delta t = 0.05 \text{ ns}\). The basic length against the dimensionless depth \(X\) and dimensionless radius \(R\) is the diameter of water drop. The influence zone of impact in the solid mainly develops in the radial direction and near to the surface. The two materials have similar traces and influence zone because of their close sonic speed.

The solution of liquid-solid impact depends strongly on the impact speed \(v_0\). With \(c_0\) the sonic speed in the undisturbed liquid, the impact Mach number is \(M_0 = v_0/c_0\). We define 3 important dimensionless parameters to characterize the main features of impact:

(i) The maximum dimensionless stress in solid, \(\Sigma_{\text{inf}}\), is the maximum dimensionless stress in solid (at all time steps before the shock wave breaks off), \(\Sigma_{\text{inf}} = \sigma_{e,max}/\).
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In this Section, we established a nonlinear wave model to solve the fully coupled liquid-solid impact problem, and detailed information of the stress field in solid and pressure field in liquid are obtained. The main features of the solutions, eqs. (27)–(31), which characterizes the peak stresses occurred during water drop-1Cr13 and Ti-6Al-V impact, are expressed in dimensionless terms. When analyzing the fatigue limit, such explicit information can be used to calculate the dimensional values of the impact stress and influence range, and then substituted into a fatigue model to evaluate the erosion process in a typical blade channel made by a typical material. If one uses a different material for the blade, similar results may be generated by following the framework in this paper.

5. Water Drop Erosion Analysis in a Typical Blade Channel

5.1 A fatigue model

Upon repetitive and high-frequency impact, the erosion process of a typical turbine blade are similar to fatique and thus water drop erosion can be explored via fatigue analysis. For water drop-1Cr13 and water drop-Ti-6Al-V impacts, the maximum impact stress and influence zone size (Section 4) can be superimposed with the statistics of impact (Section 3). Since the stress field is multiaxial and relatively complex, we use the equivalent stress as a caliber to analyze the fatigue of solid. Although the magnitude of maximum equivalent impact stress is found to be irrelevant of the diameter of water drop, the position of critical stress point scales with the diameter, which is also related with fatigue analysis. Since the sizes of water drop as well as its influence zone are much smaller than the actual blade dimension, for simplicity, we ignore the difference of the influence zone size of different particle sizes, and assume each particle would only cause damage directly beneath it. Thus, the problem of water drop erosion is transformed to the problem of fatigue cracking of the solid material under the influence of impact stress.

A simple fatigue model is established, from which the lifetime of the specimen is

$$N_i = C_i\left[\frac{\Delta \sigma_{eq}}{\Delta \sigma_{eq,h}} - \left(\frac{\Delta \sigma_{eq}}{\Delta \sigma_{eq,h}}\right)^{\frac{2}{n}}\right]^{-n/2}$$

Here, $N_i$ is the fatigue lifetime; $C_i$ is the crack resistance coefficient; $(\Delta \sigma_{eq,h})$ is the stress range of fatigue crack threshold, which is related with the fatigue limit of material. When $\Delta \sigma_{eq} \leq (\Delta \sigma_{eq,h})$, $N_i \rightarrow \infty$.

5.2 Erosion lifetime map

The material and fatigue parameters are obtained from our experiment. For 1Cr13: $(\Delta \sigma_{eq,h}) = 468$ MPa, $C_i = 24.3 \times 10^{14}$, and $n = 0.085$. For Ti-6Al-V, $(\Delta \sigma_{eq,h}) = 586$ MPa, $C_i = 24.3 \times 10^{14}$, and $n = 0.043$. Using 1Cr13 as an example, from normal impact tests, it is found that when the speed of incident water drop is below $v_d = 150$ m/s for 1Cr13, no apparent erosion could be observed experimentally. Accord-
ing to eq. (32), if the water erosion fatigue limit is set as $N_{\text{f}} = 10^5$, the threshold value of the equivalent stress range can be calculated as:

$$\Delta \sigma_{\text{eq}} \approx 468 \text{ MPa}$$

And thus the threshold of the nominal stress range are obtained from the fatigue model:

$$\Delta \sigma_n \approx 726.85 \text{ MPa}$$

We rewrite eq. (27) in dimensional form:

$$\sigma_{\text{inf}} = 0.674738 \mu A_{1,4188} \text{ (MPa)}$$

Finally, the threshold impact speed $v_{\text{th}}$ of 1Cr13 can be calculated:

$$v_{\text{th}} = 137 \text{ m/s}$$

The value predicted from our model is 9% lower than the experimental value, which should be regarded as a good agreement given uncertainties (e.g. surface roughness, water film on blade surface, stress concentration, etc.) encountered during the experiment, and thus validated that our multidisciplinary models and numerical framework could effectively capture the most dominant aspects of water drop erosion. After same procedure, the threshold impact speed $v_{\text{th}}$ of Ti-6Al-4V can be obtained as 159 m/s. It is higher than 1Cr13 because of its higher fatigue limit value.

By employing the impact velocity/distribution of water drops on the blade surface (Section 3), the erosion lifetime map turbine blade in the whole wet steam turbine stage is obtained as a function of impacted times. The result can be further translated to that represented by operating hours. With respect to eq. (13), if the entrance flux of water drop ($N_0/A_0$) is given, the impact times per hour on an area of interest ($\Delta A$) can be calculated by:

$$\Delta N = \eta_1 N_0 \Delta A/A_0 (1/\text{hour})$$

Since the number flux ($N_0$) and diameter of water drops are difficult to be measured accurately at the entrance, for convenience, an empirical coefficient $C_n$ is introduced and the lifetime counted by operating hours is ($N_t$ is the impact times to failure)

$$\Delta t = N_t / \Delta N = C_n N_t / \eta_s \text{ (hour)}$$

The value of $C_n$ can be obtained from engineering practice or experimental data. We estimate the value of $C_n$ of the target turbine blade is about $10 \times 10^5$ hour (under operating condition No. 1). Due to the value of enthalpy of steam of reduced operating condition No. 2, the number of water drops $N_0$ is estimated to be 2 times of that of No. 1.

Figure 13 shows lifetime distributions on back/inner arcs upon different operating conditions of 1Cr13. On both arcs with full load (Fig. 13(a), (b)), because the impact speed is less than the threshold speed, the minimum lifetime of whole surface is more than $40 \times 10^5$ hours. However under the operating condition No. 2 (Fig. 13(c), (d)), due to high impact speed and high impact frequency, the minimum lifetime of the surface is about $1.6 \times 10^5$ hours. For Ti-6Al-4V (Fig. 14), the corresponding value is $2.2 \times 10^5$ hours which is 37% longer than 1Cr13. For both materials, the reduced operating condition is always more dangerous, and the inner arc suffers to more severe damage than the back arc.

Finally, the most dangerous water drop erosion region and operating condition can be deduced: Under reduced load condition, the head of back arc of the blade and the band that spans from the bottom front to the middle of trailing edge are likely to suffer the most severe water drop erosion, with the second region more critical.

6. Conclusion

In this study, we developed multidisciplinary models and numerical tools to study the water drop erosion in turbine engines. Computational fluid dynamics and particle model are employed to simulate the trajectories of water drops in a flow of wet steam in the blade channel, and the most important statistics of impact are obtained. A nonlinear wave model and relevant analytical and numerical algorithms are established to explore the fundamental aspects of liquid drop-solid impact, which leads to the erosion analysis based on a fatigue model. Results are specified for water drop normal impact on two representative blade alloys, 1Cr13 and Ti-6Al-4V, and the lifetime distribution on blade surface under typical working conditions are obtained. Important conclusions include:

(1) Once the initial diameter of water drops gets larger than 50 $\mu$m, the distribution maps of impact frequency...
and velocity on the blade surface are essentially the same. In other words, the statistics of impact frequency and velocity of large water particles may be replaced by that of a fixed particle size (e.g. 50 μm), which may save computational time considerably. Regions where high impact velocity and frequency coexist may represent potentially dangerous areas for water drop erosion.

(2) The nonlinear wave model could capture the most essential characteristics of high-velocity liquid-solid impact, by ignoring the viscosity of fluid. The 3D model fully couples liquid pressure/compressibility and solid stress/deformation, and the solution focuses on the transient effects which dominate the high-velocity impact process. The transmission speed of shock wave in the radial direction is faster than that in the axial direction in the water drop. Two high transient stress regions are identified in the solid: one near the edge of the contact area and the other one near the normal axis (and several μm below the surface).

(3) The dimensionless parameters defined in this paper can well describe the impact procedure for particles of all sizes. The characteristic dimensionless scale (e.g. influence zone radius and depth), dimensionless time (e.g. time when shock wave breaks off the liquid body), dimensionless pressure (e.g. the maximum pressure at contact surface), and dimensionless stress (e.g. the maximum transient equivalent stress inside solid) are nonlinear functions of the impact velocity. Comparisons between impacts on 1Cr13 and Ti-6Al-4V are carried out, whose difference mainly arises from their different moduli. The stress patterns are close in these two materials, because their sonic speeds are close. The maximum dimensionless stress in the solid varies nonlinearly with the impact Mach number, in a form close to power-law, \( \Sigma_{inf} = 9.89132M_{inf}^{0.4188} \) for water-1Cr13 impact and \( \Sigma_{inf} = 9.12974M_{inf}^{0.4020} \) for water-Ti-6Al-4V impact. For other materials similar functions may be followed by following the same procedures using the numerical framework.

(4) With the development of a fatigue model, the most dangerous water drop erosion regions are deduced for a representative steam turbine upon typical operating conditions. The maps of erosion lifetime in terms of operations hours are obtained. Under a reduced load, the head of blade top (back arc) and the band that spans from the bottom front to the middle part of trailing edge are most critical regions for water drop erosion, which agrees with previous experimental observations. Thus, in order to alleviate water erosion, the blade should avoid to be operated under reduced load. The erosion resistance is material-dependent: although having similar sonic speed, the life time of Ti-6Al-4V is 37% longer than 1Cr13.

The analytical and numerical framework established in this paper is quite flexible and it may be extended to other liquid-solid impact/erosion problems, such as rain drop erosion.

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