Finite Element Analysis of the Onset of Necking and the Post-Necking Behaviour during Uniaxial Tensile Testing

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Tensile deformation and post-necking behaviour were investigated using the elasto-plastic finite element method. Necking at the specimen centre could be reproduced by applying a radial constraint at the loading points without assuming initial imperfections or plastic instability theory. It was found from the calculated results that for strain hardening materials Hart’s criterion overestimates the necking onset strain a little, probably due to the end effect.

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1. Introduction

The tensile test is a standard technique of mechanical testing suitable for ductile materials, which can provide quantitative measures of the intrinsic mechanical properties, such as elastic modulus, yield strength, plastic deformation, ductility and fracture. Tensile specimens of metallic materials experience elastic deformation, plastic yielding, necking instability and neck growth eventually leading to fracture due to a decrease in the load bearing capability. In order to obtain the correct mechanical properties and understand the tensile behaviour related to the intrinsic properties, not only experiments using precise machines and sensors but also interpreting the flow curves resulting from the combination of the intrinsic mechanical properties, specimen geometries and testing conditions is important.

There has been considerable progress in the researches of uniform deformation and instability necking initiation during tensile tests by the finite element method (FEM) as well as the well known analytical solutions of the criteria of Considere using a maximum load condition, eq. (1), and Hart using an area bifurcation condition, eq. (2), and a numerical method,

\[ \varepsilon_u = \frac{n}{1 + m}, \]

\[ \varepsilon_u = \frac{n}{1 - m}, \]

where \( \varepsilon_u \) is a uniform strain where necking occurs, \( n \) is a strain hardening exponent and \( m \) is a strain rate sensitivity.

Analyses of the tensile testing behaviour by FEM to date have mostly been aimed at simulating the complex physical phenomena relating to the propagation of the shear bands, fracture and shape memory materials instead of analyzing the general deformation during the tensile testing. In the simulation of the tensile testing by FEM, two schemes for selecting the calculation domain are generally used to reduce computing time instead of a calculation of the whole specimen region including a deforming zone and a grip zone: (i) a deforming region in the specimen except for the grip parts where the deformation is negligible is usually taken as a calculation domain and (ii) the geometric symmetry of the tensile specimen is considered. In adopting the former scheme, only perfectly uniform deformation occurs without necking, because every position is in the same stress state and there is no reason for a certain point to be in a higher stress state. Therefore, to prevent a perfect uniform deformation and to initiate necking during tensile testing simulations, a method of applying initial artificial imperfections to the geometry has commonly been used. However, a problem in applying the artificial initial imperfections is that a variety of results in the load, deformed geometry and stress distributions can be obtained by applying different methods for generating the various sizes, numbers and degrees of imperfections. Recently a few investigators successfully reproduced the necking phenomena by extending the calculation domain to the whole specimen including the grip parts by imposing a radial direction constraint at the tensile loading nodes, instead of applying initial imperfections. In particular, Bruenig reported that the results taking the whole specimen region as a calculation domain are almost the same as those by taking the deforming region with radial constraints. Simulating the tensile testing without initial imperfections can prevent errors depending on the characteristics of the imperfections. In addition, since most of the previous researches on tensile testing are based on assuming the initial imperfections, the previous results can be re-interpreted using the new (radial constrain) scheme. Indeed, it is valuable to investigate the validity of the new technique for reproducing the deformation behaviour in tensile testing.

Besides, as far as the authors know, there has not yet been reports for the necking instability criteria, eqs. (1) and (2), verified by numerical analyses. Therefore, it is of importance to investigate the tensile deformation and necking behaviour by use of FEM, especially for a neck initiation and a local neck growth in a post necking stage.

In this paper, finite element analysis was performed to simulate the deformation and necking behaviour during tensile testing. In particular, the effect of a strain hardening exponent on the necking onset was examined and the result was compared to the instability theory.
2. Finite Element Analysis

Isothermal two dimensional axisymmetric FEM simulations of the tensile testing have been carried out using the commercial elasto-plastic finite element code, MSC.MARC. In the tensile testing simulations, the cylindrical model specimen with the prototype geometry of a radius of 50 mm and length of 400 mm was used. One quarter of the deforming part was taken as a calculating domain, by considering the symmetries of the specimen geometry and the loading and boundary conditions, see Fig. 1. The grip part of the specimen experiencing little deformation was excluded from the calculation. It should be pointed out that the end part of the calculating domain where a nodal displacement was imposed has a radial direction constraint in order not to contract towards the centre at the grip points. This radial constraint coincides with the generally found deformed geometries of tensile specimens.

A constant velocity of 1 mm/s at the end nodes was employed until the length of the specimen reaches twice that of the initial length. The calculation stopped, being regarded as a fracture when the neck sectional area becomes 1% of the theoretical homogeneous deformation area. An isoparametric 4 nodes axisymmetric element was used, where a local adaptive mesh refinement function (h-method) in the highly deformed region was applied in order to decrease the calculation error due to a mesh distortion and in order to enhance the convergence. Using this local mesh refinement function, one element is divided three times into four smaller elements at the highly deformed elements as the strain is higher than 2; that is, one element becomes 64 elements of the 1/8 side length of the initial size. This tiny size of the elements at the highly deformed regions (neck region in this paper) has few errors due to an element size from an engineering point of view. The number of the total elements was 490 initially and increased to about 2500 at the final calculation steps.

Aiming at investigating the effect of the material properties, the strain hardening exponent \( n \) which is a major factor controlling a tensile deformation uniformity was varied. The stress–strain curves for the imaginary model materials can be expressed by the following \( n \)-th power law hardening model (Ludwik–Hollomon equation):

\[
\sigma = A \varepsilon^n + B,
\]

where \( \sigma \) is the flow stress, \( \varepsilon \) is the strain, \( \dot{\varepsilon} \) is the strain rate, \( m \) is the strain rate sensitivity and \( A \) and \( B \) are the constants. \( A = 1 \text{ MPa} \) and \( B = 0 \text{ MPa} \) were taken in this paper so as to examine the effect of only the strain hardening property.

The strain hardening exponent \( n \) was varied from \( n = 0 \) (perfect plastic or non-hardening) to 0.1, 0.2, 0.4, 0.7 and 1.0 (linear hardening). Examples of (nearly) non-hardening materials are peak-aged and over-aged Al alloys with a high dynamic recovery rate due to an easy cross slip in the peak-aged and over-aged matrix and fully pre-strained (e.g. pre-ECAP experienced) materials which show the saturation of the strength. Although the strain rate sensitivity \( m \) is an important factor affecting the post-necking behaviour until a fracture, we focused on the strain hardening effect and ignored the strain rate effect. Young’s modulus and Poisson’s ratio are set to 1000 MPa and 0.3, respectively. The time for a calculation was less than 10 min on an IBM Pentium PC.

3. Results and Discussion

Figure 2 shows the deformed geometries of the cylindrical tensile tested specimens. It is found that the present finite element calculation using the technique of imposing the radial constraint at the grip part can simulate well the necking behaviour of the tensile specimen. For a perfect plastic material of \( n = 0 \), as the tension starts, the central part of the specimen contracts faster than the other parts without an initial uniform deforming stage and the necking initiates and grows, as can be seen at the axial strain of \( \varepsilon = 0.19 \) in Fig. 2(a). Since the mesh refinement function started...
to operate at the neck region where the effective strain exceeded two, a very fine mesh system at the neck region
could correctly represent the highly deformed outline profile
as well as the internal region. At the necking initiation points
e in the neck region. Figure 2 shows that the gradient (dr/dz)
of the neck outer profile decreases with n, by examining
the specimen geometries of n = 0.1, 0.2 and 0.4 respectively, there are no notable local deformations in
the outline profiles of the specimens or mesh distortions
in the neck region. Figure 2 shows that the gradient (dr/dz)
of the neck outer profile decreases with n, by examining
the specimen geometries of n = 0.1, 0.2 and 0.4 cases at
the fracture points; the fracture in the simulations was defined
in this paper as the strain of $\varepsilon = 0.283, 0.368$ and $0.523$,
respectively, where (neck sectional area)/(sectional area
in perfect uniform elongation) = 1%. Here r and z are the
radial and axial coordinates, respectively. This smooth neck
profile, i.e. more uniform deformation, at the specimen of
a higher n material is attributed to the higher local
deformation, higher local strength due to strain hardening
and the diffusion of the deformation at the neck
when compared to the neck’s outside region. The linear hardening
material of n = 1.0 is uniformly elongated without a necking,
except near the grip region, until the length becomes 2.25
times ($\varepsilon = 0.81$) of the initial length.

As the tensile testing proceeds, a sectional area at the neck
varies. Figure 3 represents the relative neck area normalised
by the neck area in an ideally uniform elongation. The dotted
curves correspond to the theoretical uniform elongation,
where the uniform sectional area $A_u$ decreases almost linearly
with the tensile strain as derived in eq. (4).

$$A_u = \frac{A_0 l_0}{l} = A_0 e^{-\varepsilon} \approx A_0 (1 - \varepsilon).$$

where $A_0$, $l_0$ and $l$ are the initial sectional area, initial length
and elongated length of the tensile specimen, respectively.

In Fig. 3 it can be found that i) the neck area of the linear
strain hardening material of $n = 0$ shown as dashed lines
decreases similar to the uniformly deforming case and ii) the
neck area of the perfect plastic material of $n = 1$
decreases rapidly from the initial stage, deviating from
the uniformly deforming curve. The strain hardening materials

0 < n < 1 represent a uniform elongation, necking and post-
necking behaviour sequentially, and it can be ascertained that
a uniform elongation is prolonged as n increases. An
interesting point in Fig. 3 is that not only the sectional area
reduction behaviour in the uniform elongation stage is
the same but also the gradients of the curves in the post-
necking stages are very similar in all the materials of different
n values.

In order to compare the post-necking behaviour shown
in Fig. 3, the neck area vs. axial strain curves of $n = 0.1,
0.2$ and $0.4$, shifted along (−) axial strain direction by the
necking initiation strains ($\varepsilon_u = 0.084, 0.174$ and $0.345$,
respectively) are drawn in Fig. 4. These shifted post-necking
curves of the neck area coincide well with the curve of a
perfect plastic material. This coincidence of the post-necking
curves means that the neck growth process is not affected
much by the material’s intrinsic strain hardening property,
because the strain is high, the stress becomes saturated,
the strain hardening is very low and the flow curves approach
perfect plastic curves in the neck region, see Fig. 5(a).

Figure 5 is the stress–strain curves based on (a) the
Ludwik–Hollomon eq. (3), (b) apparent engineering values
calculated by FEM and (c) apparent true values calculated
by FEM. Here ‘apparent’ means the averaged values based
on the length of the specimens and the total load. Figs. 5(a)
and (b) are the calculated engineering and true stress–strain
curves showing the elastic, plastic hardening, instability
necking and post-necking before a fracture, which can be
generally found in ductile metallic materials. It was difficult
to explicitly determine the neck initiation point from the
mesh deformation (Fig. 2) or neck area variation (Fig. 3),
although it is clear that a uniform elongation increases with
the strain hardening exponent n. Hence, a uniform elonga-
tion, i.e. necking initiating point, was determined from the
calculated load maximum point in Fig. 5(b) similar to the
Considere’s approach, instead of from the deformation
geometries. The uniform elongation values measured
by FEM, $\varepsilon_u = 0.084, 0.174$ and $0.345$ for $n = 0.1, 0.2$ and
0.4 respectively, are lower than the theoretical values of the
Considere’s criterion (which is the same as the
Hart’s criterion in rate insensitive materials), \( u = 0, 0.1, 0.2 \) and 0.4. This discrepancy is attributable to the end effect of the specimens, where the deformation is constrained by the grip. Therefore we can expect that the discrepancy will increase in shorter specimens.

Figure 6 is the effect stress distribution at various deformation stages of the tensile testing. At the necking initiation point (e.g. \( n = 0.4, \varepsilon = 0.345 \)) the stress distribution is uniform except in the grip end region. It is interesting that the materials with different hardening properties show almost the same stress values and distributions in the neck regions at the post-necking stages of the same (neck area)/(uniform neck area) states, \( u = 0.190, 0.368 \) and 0.523 for \( n = 0, 0.2 \) and 0.4, respectively. This is because of the stress saturation in the local neck region and related to the coincidence of the post-necking curves in Fig. 4.

The other regions out of the neck areas after necking become elastic states due to the stress release during the necking.

4. Conclusions

An elasto-plastic finite element analysis was performed to investigate the deformation and necking behaviour of specimens during tensile testing. The necking initiation and post-necking could be simulated well by the use of the radial direction constraint at the grip part without imposing initial imperfections. The calculations demonstrated the strain hardening dependency of the tensile deformation procedure; the perfect plastic material of \( n = 0 \) started its neck growth stage from the beginning of the tensile testing, the linear strain hardening material of \( n = 0.4 \) deformed uniformly at least until the axial tension of 125% and the strain hardening materials of \( 0 < n < 1.0 \) showed the elastic deformation, uniform plastic deformation, necking initiation and neck growth stages. The uniform elongation strain where the instability necking was initiated was lower than the theoretical values of Considere and Hart’s theoretical criterion \( (\varepsilon_{u} = n) \), probably due to the end effect. The neck growth behaviour of the materials coincides with that of the perfect plastic material, irrespective of the strain hardening exponent \( n \). From the current results, it was possible to better understand the tensile testing curves. One of the important factors in tensile necking is strain rate effect. The strain rate effect associated with the strain rate sensitivity is in progress and the results will be presented separately.

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