**Prediction of the Occurrence of Swirl Motion at Interface between Stratified Two Liquids**

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An interface between stratified two liquids contained in a cylindrical vessel rotated around the vertical vessel axis when a bottom blown jet of the lower liquid impinged on the interface. This swirl motion has high mixing intensity, and hence, is useful for enhancing mass transfer at the interface. A critical condition for the occurrence of the swirl motion was experimentally investigated. Empirical equations proposed originally for predicting the critical occurrence condition of a swirl motion in a single-liquid bath were applicable to this case by replacing the gravitational acceleration constant by a modified gravitational acceleration one.

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### 1. Introduction

Recently the present authors have proposed a novel agitation method for enhancing the efficiencies of metals refining processes, extraction processes, and wastewater treatment using a swirl motion of a bottom blown liquid jet.¹⁻³) As a model study, a liquid was injected into a bath of the same liquid contained in a cylindrical vessel through a centered bottom nozzle with a pump, as shown schematically in Fig. 1. The liquid was drained through four holes settled on the bottom plate of the vessel and injected into the bath again. Namely, the liquid was circulated to keep the bath depth constant. The jet rotated around the vertical vessel axis under a certain condition. Mixing time in the bath was significantly shortened in the presence of the swirl motion.⁴) This swirl motion is very similar to the rotary sloshing appearing in a cylindrical bath oscillated externally in the vertical or horizontal direction (see Fig. 2).⁵,⁶) In fact, the period of the swirl motion was satisfactorily predicted by an analytical solution for the period of the rotary sloshing.⁵)

A swirl motion also occurs when another liquid is placed on the bath.⁷) Two types of swirl motions were observed depending on the thickness of the upper layer (see Fig. 3). When the thickness of the upper layer was smaller than a certain critical value, the upper layer rotated together with the lower layer. This type of swirl motion was tentatively named Type A. On the other hand, when the upper layer was thicker than the critical value, a swirl motion appeared at an interface between the two liquid layers. This swirl motion behaved as if the gravitational acceleration were much smaller than 9800 mm/s². The period of the swirl motion was predicted by an analytical solution of the rotary sloshing by replacing the gravitational acceleration, $g$, by the following modified gravitational acceleration.⁷,⁸)

$$g' = \frac{(\rho_{L2} - \rho_{L1})g}{(\rho_{L1} + \rho_{L2})}$$

(1)

where $\rho_{L1}$ and $\rho_{L2}$ denote the densities of the upper and lower liquids, respectively. $(\rho_{L2} - \rho_{L1})/(\rho_{L1} + \rho_{L2})$ is called the stratification parameter. This swirl motion is very similar to the above-mentioned rotary sloshing and tentatively called Type B.

The conditions under which the two types of swirl motions occur are not fully understood yet. In this study attention was paid only to Type B because it is useful for enhancing the...
mass transfer between stratified two liquids. The occurrence condition for Type B was experimentally investigated based on cold model experiments. Discussion was given on the applicability of empirical equations proposed for predicting the occurrence region of the swirl motion of a single-liquid jet\(^9\) to this case.

Although injection of the upper liquid into the lower liquid is expected to enhance mixing of the two liquids, droplets of the upper liquid are likely to occur compared to the lower liquid injection and the droplets are accumulated near the interface. As a result, wave motions are suppressed by the droplets. This process will be investigated in a future study.

2. Experimental Apparatus and Procedure

Figure 4 shows a schematic of the experimental apparatus.\(^7\) The cylindrical test vessel was made of transparent acrylic resin. At first, water was filled into the vessel to a predetermined depth. Kerosene, n-pentane, or silicone oil was placed on the water bath. The water in the bath was drawn from four drainage holes settled on the bottom plate of

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**Fig. 3** Two types of swirl motions appearing in a bath of stratified two liquid layers.

**Fig. 4** Experimental apparatus.
the vessel and injected from a centered single-hole bottom nozzle into the bath. Namely, the water was circulated with a pump to keep the bath depth constant. The positions of the drainage holes are 47.5 mm and 50 mm apart from the centerline of the vessel for \( D = 130 \text{ mm} \) and 150 mm, respectively. These positions did not affect the swirl motion as long as they were placed near the side wall of the vessel, as in the case of a single-liquid.\(^2\) The test vessel was enclosed with another vessel with a rectangular cross-section made of transparent acrylic resin, although the rectangular vessel was not drawn in order to avoid crowding in Fig. 4. The clearance between the rectangular vessel and the cylindrical vessel was filled with water to avoid distortion of video images of a swirling interface taken from the side of the vessel.

On the other hand, fluorinert was chosen as the lower liquid because its density is greater than that of water. Water was placed on the fluorinert layer, and the fluorinert was circulated. The physical properties of the liquids used in this study are listed in Table 1. The vessel diameter, thickness of the upper layer, thickness of the lower layer, inner diameter of the injection nozzle, and water flow rate were represented by \( D \), \( H_{L1} \), \( H_{L2} \), \( d_{nen} \), and \( Q_L \), respectively.

The occurrence of a swirl motion at the interface was judged with a digital video camera and by eye inspection. The measurements were repeated more than two times under every experimental condition to determine the condition for the occurrence of the swirl motion of Type B. A period from the start of liquid injection to the moment at which the swirl motion attains a steady state, named the starting time of swirl motion, should be referred to the previous paper.\(^3\)

The experimental results presented in the previous study \(^7\) are briefly explained again for better understanding of a swirl motion of Type B. The occurrence condition of the swirl motion was not affected by the thickness of the upper layer as long as the aspect ratio of the upper layer, \( \frac{H_{L1}}{D} \), was greater than 0.5. A swirl motion of Type A was observed for \( \frac{H_{L1}}{D} < 0.5 \). The boundary of the region in which a swirl motion of Type B appeared was found to be correlated as a function of the aspect ratio of the lower layer, \( \frac{H_{L2}}{D} \), and the following modified Rossby number, \( \text{Ro}_{m}^{'}. \)

\[
\text{Ro}_{m}^{'} = \frac{Q_L^2}{(g'd_{nen}^2D^5)}
\]

where \( g' \) is the modified gravitational acceleration expressed by eq. (1). The physical properties of liquids (kinematic viscosity and density) did not affect the boundary. The interfacial tension between two liquids was approximately 55 mN/m in every case. Effect of the interfacial tension on the occurrence condition of a swirl motion must be left for a future study. In addition, the boundary was divided into four sub-boundaries, (1), (2), (3), and (4), as schematically shown in Fig. 5, just like the boundary for the swirl motion of a single-liquid jet.\(^9\) In the following the functional relationship between \( H_{L2}/D \) and \( \text{Ro}_{m}^{'} \) will be derived for each sub-boundary.

### 3.2 Relationship between \( H_{L2}/D \) and \( \text{Ro}_{m}^{'} \) for sub-boundary (1)

As mentioned in the previous section, the swirl motion observed in this study resembles the rotary sloshing occurring in a cylindrical vessel subjected to horizontal or vertical forced oscillation.\(^5,6\) Referring to the previous paper,\(^9\) we assume that the swirl motion around the sub-boundary (1) occurs when the elevated height of the interface averaged over the cross-section of the vessel, caused by impingement of a jet of the lower liquid, \( H_{L2}(t) \), exceeds a certain critical value. This condition is expressed by

\[
H_{L2}(t)/D \geq k_1
\]

\[
\text{Ro}_{m}^{'} = [Q_L/(\pi D^2/4)]^2/(2g')
\]

\[
Q_L = 0.3Q_l,H_{L2}/d_{nen} \quad (100 \geq H_{L2}/d_{nen} \geq 3.3)
\]

\[
Q_l = Q_l
\]

\[
H_{L2}/d_{nen} < 3.3
\]

These expressions yield

\[
H_{L2}/D \geq m_1(\text{Ro}_{m}^{'})^{-1/2}
\]
The coefficient, \( m_1 \), was tentatively assumed to be the same as that for a single-liquid bath. Accordingly, the sub-boundary (1) is expressed by

\[
H_{1,2}/D = 0.0131(Re_m)^{-1/2}
\]

(7)

The most adequate value of \( m_1 \) in the present case will be determined later by fitting eq. (6) to measured values of the aspect ratio indicating the sub-boundary (1).

3.3 Relationship between \( H_{1,2}/D \) and \( Re_m \) for sub-boundary (2)

We assume that the swirl motion appears on the sub-boundary (2) when the pressure fluctuation at the interface exceeds a certain critical value.

\[
\rho_{L2}g\left[Q_{L2}/(4\pi b_u^2)\right]^2/(2g)/(\rho_{L2}gH_{1,2}) \geq k_2
\]

(8)

\[
b_u = 0.09H_{1,2}
\]

(9)

where \( \rho_{L2} \) is the density of the lower liquid, \( b_u \) is the half-value radius of a liquid jet at the interface, and \( k_2 \) is a constant. In eq. (8), \( \rho_{L2}gH_{1,2} \) is the initial static pressure on the bottom wall of the vessel. Substitution of eqs. (5a) and (9) into eq. (8) gives

\[
H_{1,2}/D \leq m_2(Re_m)^{-1/3}
\]

(10)

If a relationship for a single-liquid bath\(^9\) is also valid in this case, the sub-boundary (2) is described by

\[
H_{1,2}/D = 6.24(Re_m)^{1/3}
\]

(11)

3.4 Relationship between \( H_{1,2}/D \) and \( Re_m' \) for sub-boundary (3)

The swirl motion is assumed to cease when the diameter of a liquid jet on the interface exceeds a certain critical value\(^9,11\)

\[
D_{jet}/D > k_3
\]

(12)

where \( k_3 \) is a constant. The diameter of the jet at the interface, \( D_{jet} \), can be approximated by

\[
D_{jet} = 4b_u
\]

(13)

where \( b_u \) is the half-value radius expressed by eq. (9). Referring to the experimental results for a single-liquid bath\(^9\), we have

\[
H_{1,2}/D > 1.69
\]

(14)

The sub-boundary (3) is expressed by

\[
H_{1,2}/D = 1.69
\]

(15)

3.5 Relationship between \( H_{1,2}/D \) and \( Re_m' \) for sub-boundary (4)

In the previous study\(^9\) it was difficult to clearly judge the cessation of the swirl motion around the sub-boundary (4). This is because violent splashing is imposed on the swirl motion. As a result, the scattering of the measured values of the aspect ratio indicating the sub-boundary (4) became large, and hence, two empirical equations were proposed, as shown in the following.

3.5.1 Relationship based on the inertial force of injected liquid

When the inertial force of the injected liquid, \( F_i2 \), becomes greater than a certain critical value, the jet is likely to rise straight upward and, as a result, the swirl motion is considered to cease.

\[
F_i2/W_{LB2} > k_4
\]

(16)

\[
F_i2 = \rho_{L2}Q_{L2}^2/\pi d_{net}^2/4
\]

(17)

\[
W_{LB2} = \rho_{L2}gH_{1,2}^2\pi D^2/4
\]

(18)

where \( W_{LB2} \) is the weight of the lower liquid in the bath and \( k_4 \) is a constant. Substitution of eqs. (17) and (18) into eq. (16) yields

\[
H_{1,2}/D < m_4Re_m'
\]

(19)

By referring to the swirl motion in a single-liquid bath\(^9\), the sub-boundary (4) is expressed by

\[
H_{1,2}/D = 26Re_m'
\]

(20)

3.5.2 Relationship based on the height of locally elevated interface

We assume that the swirl motion ceases when the height of the locally elevated interface exceeds a certain critical value\(^9\)

\[
H_{1,2}/D = 3.6(Re_m')^{1/2}
\]

(21)

3.6 Comparison of empirical equations with measured results

3.6.1 \( \text{N-pentane}/\text{water bath} (\rho_{L1}/\rho_{L2} = 0.63 \text{ and } v_{L1}/v_{L2} = 0.41) \)

N-pentane was chosen because its density and kinematic viscosity are much lower than their respective values of water. In Fig. 7 the aspect ratio, \( H_{1,2}/D \), indicating the sub-boundary (1) for \( D = 150 \text{ mm} \) and \( d_{net} = 5 \text{ mm} \) is much greater than that in a single-liquid bath. Accordingly, eq. (7) was modified in the following form by taking experimental results for other liquids shown later into consideration.

\[
H_{1,2}/D = 0.050(Re_m')^{-1/2}
\]

(22)

The sub-boundaries (2) and (3) shown in Fig. 7 can be approximated by eqs. (11) and (15), respectively. Equation (22) is not drawn to avoid crowding in the figure. The sub-boundary (4) falls around eqs. (20) and (21).
At present, a clear explanation cannot be given on the difference between eqs. (7) and (22). Further investigation is required on the sub-boundary (1).

### 3.6.2 Kerosene/water bath ($\rho_{L1}/\rho_{L2} = 0.79$ and $v_{L1}/v_{L2} = 1.54$)

Kerosene was used because its density and kinematic viscosity are close to their respective values of water. The measured values indicating the boundary of the occurrence region of a swirl motion for $D = 150$ mm and $d_{nen} = 5$ mm are shown in Fig. 8. Empirical equations, (22), (11), and (15) are applicable to the sub-boundaries (1)–(3) for the kerosene/water bath as well. The sub-boundary (4) is located around eqs. (20) and (21), just as mentioned for a single-liquid bath.9)

### 3.6.3 Fluorinert/water bath ($\rho_{L1}/\rho_{L2} = 0.59$ and $v_{L1}/v_{L2} = 2.27$)

Fluorinert was chosen to investigate the effect of the density of the lower liquid on the swirl motion. Water was placed on the fluorinert layer. Figure 9 shows that the sub-boundaries (1)–(3) for $D = 150$ mm and $d_{nen} = 5$ mm can be approximated by eqs. (22), (11), and (15), respectively. The sub-boundary (4) is not uniquely approximated by eq. (20) nor by eq. (21) just as in the preceding cases.

### 3.6.4 Silicone oil/water bath

Two kinds of silicone oils were chosen because the kinematic viscosity of the upper liquid, $v_{L1}$, can readily be increased and the density difference can also be significantly decreased.

1. Silicone oil of $v_{L1} = 1.0$ mm$^2$/s ($\rho_{L1}/\rho_{L2} = 0.82$ and $v_{L1}/v_{L2} = 1.10$)

   Figure 10 compares the observed boundaries of the occurrence region of the swirl motion for a silicone oil of $v_{L1} = 1.0$ mm$^2$/s with the empirical equations derived in this study. The aspect ratio of the upper layer, $H_{L1}/D$, was largely varied. The sub-boundary (1) is roughly approximated by eq. (22). The sub-boundaries (2) and (3) are satisfactorily predicted by eqs. (11) and (15), respectively. The sub-boundary (4) is located around eq. (20).

   The experimental results for $D = 150$ mm and $d_{nen} = 5$ mm are shown for three different nozzle diameters in Fig. 11. The sub-boundaries (1) through (3) also were approximated by eqs. (22), (11) and (15), respectively. The measured values indicating the sub-boundary (4) are largely scattered between eqs. (20) and (21) just like that boundary in a single-liquid bath. Such a degree of scattering is attributable to violent splashing imposed on the swirl motion and the droplet formation at the interface, as mentioned earlier.

   2. Silicone oil of $v_{L1} = 10$ mm$^2$/s ($\rho_{L1}/\rho_{L2} = 0.94$ and $v_{L1}/v_{L2} = 11.0$)

   Figure 12 shows the measured values of the aspect ratio for the boundary of the occurrence region of a swirl motion for silicone oil of $v_{L1} = 10$ mm$^2$/s. The aspect ratio of the upper layer, $H_{L1}/D$, was varied just as shown in Fig. 10. The bath diameter was $D = 130$ mm and the inner diameter of the
nozzle, \(d_{\text{nen}}\), was 5 mm. The occurrence region became very narrow. This is because many water droplets are generated near the interface and trapped at the interface. The droplet formation is closely associated with a very small density difference, \(C_26 / C_26 L_2 / C_26 L_1\), of 61 kg/m\(^3\).

Figure 13 shows the boundary of the occurrence region of a swirl motion for \(D = 150\) mm and \(d_{\text{nen}} = 5\) mm. The occurrence region became narrow as the kinematic viscosity of silicone oil increased. In addition, the occurrence region in Fig. 13 is much different from that shown in Fig. 12. This is explained by the fact that the generation of water droplets near the interface is also dependent on the vessel diameter, \(D\).

The results shown above mean that when the density ratio, \(\rho_{L1}/\rho_{L2}\), falls between 0.59 and 0.82, the sub-boundaries (1)\textsuperscript{1}-(3) can be satisfactorily predicted by eqs. (22), (11), (15), respectively. The measured values of the aspect ratio indicating the sub-boundary (4) are relatively scattered and distributed around eqs. (20) and (21). At present, it is difficult to conclude which empirical equation is more adequate, eq. (20) or (21). Further investigation also is required for the sub-boundary (4).

4. Conclusions

Investigation was made of a swirl motion appearing at an interface between stratified two liquid layers contained in a cylindrical vessel. A bottom blown jet of the lower liquid impinged onto the interface. A swirl motion appeared at the interface when the thickness of the upper layer was greater than a certain critical value \(H_{L1}/D > 0.5\). The boundary of
the region in which the swirl motion occurs can be correlated as a function of the aspect ratio of the lower layer, $H_{L2}/D$, and the modified Rossby number, $R_0m'$. The effects of the physical properties of liquids on the boundary are negligibly small for $\rho_{L1}/\rho_{L2} = 0.59\sim0.82$ and $\nu_{L1}/\nu_{L2} = 0.41\sim2.27$. The boundary can be divided into four sub-boundaries. Empirical equations were proposed for describing the sub-boundaries as functions of $H_{L2}/D$ and the modified Rossby number, $R_0m'$, based on a modified gravitational acceleration, $g'$.

Nomenclature

- $b_u$: half-value radius of liquid jet (mm)
- $D$: vessel diameter (mm)
- $D_{jet}$: diameter of liquid jet at the interface (mm)
- $d_{cen}$: nozzle diameter (mm)
- $F_{I2}$: inertial force of injected liquid (N)
- $g$: gravitational acceleration (mm$^2$/s)
- $g'$: modified gravitational acceleration (mm$^2$/s)
- $H_{L1}$: total bath depth ($=H_{L1}+H_{L2}$) (mm)
- $H_{L1}$: thickness of upper layer (mm)
- $H_{L2}$: thickness of lower layer (mm)
- $H_{LeB}$: height of elevated interface due to impingement of liquid jet (mm)
- $Q_{L}$: liquid flow rate at nozzle exit (mm$^3$/s)
- $Q_{Ls}$: liquid flow rate at interface (mm$^3$/s)
- $R_0m'$: modified Rossby number based on $g'$ (~)
- $W_{LB2}$: weight of lower liquid in bath (N)
- $\nu_{L1}$: kinematic viscosity of upper liquid (mm$^2$/s)
- $\nu_{L2}$: kinematic viscosity of lower liquid (mm$^2$/s)
- $\rho_{L1}$: density of upper liquid (kg/m$^3$)
- $\rho_{L2}$: density of lower liquid (kg/m$^3$)

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