Modeling of Thermo-Mechanical Stresses in Twin-Roll Casting of Aluminum Alloys

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A two-dimensional steady state thermo-mechanical stress model was developed using the commercial structural analysis package ANSYS, to compute the stresses arising from thermal gradients and mechanical loads applied to strip and roll during a horizontal twin-roll thin strip casting process. The finite element mesh and the nodal temperature values obtained from the fluid flow, heat transfer, and solidification model developed on FIDAP were transported to ANSYS. Mechanical loads like roll separating force and strip exit tension were also applied on the strip. A visco-plastic constitutive relation has been used to describe the behavior of solidifying aluminum alloy. The effects of inlet velocity of the melt and contact strip/roll heat transfer coefficient on the resultant stress profile in the strip and the rolls were investigated.

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Keywords: mathematical modeling, thin strip casting, thermo-mechanical stresses, visco-plasticity

1. Introduction

A major problem encountered in twin-roll casting process is the formation of cracks and other defects in the solidified strip. These defects arise due to thermo-mechanically induced stresses and strains due to temperature gradients, applied pressures, and loads. It is of great importance to understand the evolution of these stresses and strains and formation of defects in the cast strip due to these stresses. There have been significant advances in the numerical simulation of casting and solidification process. Numerous models for coupled fluid flow, heat transfer, and solidification in thin strip casting processes have been developed. But very few papers have dealt with modeling of stresses in thin strips.

Modeling of stresses in DC casting of aluminum1–5) has received much attention of the researchers in the past. Hannart et al.3) developed a fully coupled, three dimensional, transient FEM model assuming the metal to be elasto-plastic with strain-rate dependent behavior. The model estimates the temperature and thermal stress evolution during DC casting of aluminum slabs. Fjaer and Mo2) developed a mathematical model in which the thermally induced strains and stresses, which develop during direct-chill semi-continuous casting of aluminum billets, were calculated by finite element method. They analyzed the effect of casting speed on the thermal and residual stresses. In another paper by Fjaer and Mo,3) they developed a mathematical model in which the metal was assumed to be an isotropic elastic-viscoplastic material with strongly temperature-dependent properties. Apart from calculation of thermally induced strains and stresses, they also studied the development of plastic yield and the development of an air gap between the billet and the bottom block. Drezet and Rappaz4) developed a two-dimensional transient thermo-mechanical model to ingot deformation during direct chill casting (DC) and electromagnetic casting (EMC) of aluminum alloy slabs. Flood et al.5) concluded that non-linear elastic stress calculations are required to predict the occurrence of cold cracks in DC casting of aluminum alloys.

Few papers have directly dealt with mathematical modeling of stresses and strains in the twin roll thin strip casting processes. Mo and Hoydal6) developed a two-dimensional mathematical model to calculate the stresses within the horizontal thin strips. They assumed the problem to be plain strain by neglecting the edge effects. They found that shear deformation was severe both close to the solidification and close to the exit. Jarry et al.7) proposed a thermo-mechanical model for 3C (Continuous Casting between Cylinders) roll casting of alloys. They considered the coupling between the thermomechanics of solidification and mechanics of deformation due to rolling operation. The contact pressure between the strip and the rolls was shown to be the central variable, which governs both, heat transfer and stress/strain generation in a totally coupled manner.

In the present work, a two-dimensional finite element model was developed to calculate the stress and strain field in the solidified aluminum alloy during horizontal twin-roll thin strip casting process. The model uses a viscoplastic constitutive equation to describe the material behavior at elevated temperatures. The temperature profile obtained from the thermal and fluid flow calculations performed using commercial software FIDAP was imposed as a thermal load on the system. The fluid flow, heat transfer and solidification model is described in a previous paper by the authors.8) The stress calculations were performed by a model based on commercial software ANSYS.

2. Mathematical Model

Unlike the thin strip casting of steel, the solidification in case of aluminum alloys casting completes before the kissing point of the rolls. Hence the strip undergoes considerable reduction after completing solidification. Stresses in thin strip casting primarily arise due to:

- Thermal gradients in the material, which results in very high thermal stresses in the solidifying material.
The system.

Thermal strain vector is given by:

\[ \sigma_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p + \epsilon_{ij}^{Th} \]  

where, \( \sigma_{ij} \) are the Cartesian components of the Cauchy stress tensor in the body and \( f_i \) represents the body force acting on the system. 

The total strain tensor, \( \epsilon_{ij} \) can be decomposed in the following way:

\[ \epsilon_{ij} = \epsilon_{ij}^e + \epsilon_{ij}^p + \epsilon_{ij}^{Th} \]  

where, \( \epsilon_{ij}^e \) is the elastic, \( \epsilon_{ij}^p \) is the plastic, and \( \epsilon_{ij}^{Th} \) is the thermal part, respectively.

According to the constitutive law for isotropic linear elasticity, the stress is related to strains by:

\[ \sigma_{ij} = E_{ijkl}(T) \epsilon_{kl}, \quad i, j, k, l = 1, 2, 3 \]  

Thermal strain vector is given by:

\[ \{ \epsilon_{ij}^{Th} \} = \Delta T (\alpha, \alpha_0, 0, 0, 0, 0)^T \]  

Inelastic deformation, \( \epsilon_{ij}^p \), was calculated according to a rate dependent viscoplastic constitutive model.

Anand’s model \(^9\) for viscoplasticity was employed in the present work. There are two basic features in Anand’s model applicable to isotropic rate-dependent constitutive model for metals. First, there is no explicit yield surface; rather the instantaneous response of the material is dependent on its current state. Secondly, a single scalar internal variable, \( \delta \), called the deformation resistance, is used to represent the isotropic resistance to inelastic flow of the material. The specifics of the constitutive equation are described elsewhere. \(^9\) The numerical values of the parameters used in this model are given in Table 1.

### 2.2 Special procedure for liquid regions

Since both liquid and solid regions are part of the calculation domain, a special procedure was employed to handle the liquid region. \(^{10} \) A value of Poisson’s ratio very close to 0.5 is assigned at the nodes where temperature is above the coherence (or zero-strain) temperature. This makes the liquid phase close to being incompressible for mechanical loading. In order to avoid singularity in forming the stiffness matrix, the Young’s modulus is set to a very small number, instead of exactly zero, at the nodes above the coherence temperature.

For a non linear elastic solid,

\[ \sigma_{ij} = \sigma_{ij}^p + p \delta_{ij} \]  

where,

\[ \sigma_{ij}^p = 2G \epsilon_{ij} \]  

\[ p = \lambda \epsilon_v - \kappa (T - T_{ref}) \]  

where,

\[ G = \frac{E}{2(1 + v)} \]  

\[ \lambda = \frac{E v}{(1 + v)(1 - 2v)} \]  

\[ \kappa = \frac{E \alpha}{3(1 - 2v)} \]  

Therefore, when the Poisson’s ratio \( v \) is set close to 0.5 and Young’s modulus is a very small number, the deviatoric stress \( \sigma_{ij}^p \) can be suppressed whilst keeping the hydrostatic pressure finite.

### 3. Results and Discussion

#### 3.1 Calculation of roll separating force

Since the process of thin strip casting involves not only solidification but rolling too, calculation of rolling force is an important parameter to be studied. In case of aluminum alloys, the solidification takes well before the kissing point (i.e.
least separation) of the rolls; hence the solidified strip undergoes a lot of rolling before it exits. Strictly speaking the rolling action starts as soon as the layer of the melt in contact with the rolls solidify. But the rolling separating force involved is very low till complete solidification has taken place. In the present study, the total roll separating force was calculated, starting from the point of complete solidification to the exit point of the strip for different conditions of strip/roll heat transfer coefficient and inlet velocity. Sticking friction the exit point of the strip for different conditions of strip/roll calculated, starting from the point of complete solidification to the exit point of the strip for different conditions of strip/roll heat transfer coefficient and inlet velocity. Sticking friction the exit point of the strip for different conditions of strip/roll heat transfer coefficient and inlet velocity. Sticking friction

\[
R.F. = 1.55 \sigma_{\text{yield}} W \left[ 1 + \frac{\sqrt{R \Delta h}}{4 \left( h_1 - \left( \frac{h_{\text{mean}}}{2} \right) \right)} \right] \sqrt{R \Delta h} \tag{11}
\]

where,

\[
h_{\text{mean}} = \left( h_1 - \left( \frac{\Delta h}{2} \right) \right) \tag{12}
\]

and

\[
\Delta h = h_2 - h_1 \tag{13}
\]

The data for variation of mean yield stress with temperature for rolled aluminum rod, given in Ref. 12) has been used. The total rolling force was calculated for different conditions of operating variables and is presented in Fig. 2. The rolling force was calculated for only those cases in which the complete solidification takes place before the kissing point of the rolls. The rolling force is of the order of 10^6 N for low inlet velocities of 1 and 5 mm/s. Strip/roll heat transfer coefficient has a little effect on the resulting roll force as compared to the effect of inlet velocity. At high inlet velocity the roll force is independent of heat transfer coefficient and remains nearly constant on variation of the heat transfer coefficient.

### 3.2 Stress analysis of strips

The roll force values obtained above were applied to the strip and the reaction forces on the roll itself. A constant value of strip exit tension of 20000 N was applied for all the cases. The simulation was performed for different values of strip/roll heat transfer coefficient and inlet velocity to study the effect of these variables on the resulting stress profile. The melt superheat was taken to be constant equal to 30 K. An inlet velocity of 10 mm/s and a contact strip/roll heat transfer coefficient of 10000 W/m^2K were taken as the base case.

Figure 3 shows the \( \sigma_x \) stress profile within the strip for inlet velocity of 10 mm/s and contact strip/roll heat transfer coefficient of 10000 W/m^2K. The region near the inlet is stress free or the values of stresses in this region are close to zero owing to temperature greater than the solidus temperature. The reference temperature in this region was taken to be the solidus temperature (842 K). Stresses, at the region between the inlet and the roll surfaces are tensile in nature. This is due to presence of very high thermal gradients as the melt gets solidified by coming in contact with the roll surface. The strip is cooled due to the quenching effect of the rolls, hence the average roll surface temperature of the surface of roll in contact with the strip was taken as the reference temperature for all the regions within the strip where the temperature is less than the solidus temperature. As the strip gets solidified, \( \sigma_x \) gradually become compressive in nature. Stresses at the strip exit point are tensile in nature because of the strip exit tension applied.

Figure 4 shows the \( \sigma_y \) stress profile within the strip for inlet velocity of 10 mm/s and contact strip/roll heat transfer coefficient of 10000 W/m^2K. As can be seen in the Fig. 4, stresses in the region near to inlet are close to zero. The roll force is applied only after complete solidification has taken place. The \( \sigma_y \) stresses in region halfway to the kissing point are compressive in nature. This is due to the combined effect of thermal gradients within the strip and the roll force transmitted to the strip. At the strip exit point, the average temperature across the through thickness of the strip is less than the reference temperature (i.e. the average roll surface temperature). The magnitude of tensile stresses produced due to this temperature difference is greater than the compressive stresses produced due to roll force transmitted.

Figure 5 shows the Von-Mises equivalent stress profile within the strip for inlet velocity of 10 mm/s and contact strip/roll heat transfer coefficient of 10000 W/m^2K. Von-Mises equivalent stress profile within the strip for inlet velocity of 10 mm/s and contact strip/roll heat transfer coefficient of 10000 W/m^2K.
Modeling of Thermo-Mechanical Stresses in Twin-Roll Casting of Aluminum Alloys

3.2.1 Effect of strip exit tension

Figure 6 shows the variation of \( \sigma_x \) stresses across the through thickness of strip at strip exit point for different values of strip exit tension and at constant contact strip/roll heat transfer coefficient of 10000 W/m²K and constant inlet velocity of 10 mm/s.

Simulations were performed for strip exit tension values of 0, 10000, and 20000 N. Strip exit tension was applied in the x-direction and so it did not affect the values of \( \sigma_y \) stresses. When there is no strip exit tension applied the \( \sigma_x \) stresses at the strip exit point are compressive stresses of lower magnitude (i.e., average value of \(-4 \text{ MPa}\)). When a strip exit tension of 10000 N is applied, the \( \sigma_x \) stresses are all tensile in nature with average magnitude of 17 MPa and maximum stress of 18.5 MPa at the center of the strip exit region. Increasing the strip exit tension to 20000 N increases the average magnitude of tensile stresses to 35 MPa and the maximum stress to 41 MPa at the center of the strip exit region.

3.2.2 Effect of inlet velocity

Inlet velocity also affects the resultant stress profile considerably. Von-Mises equivalent stress profiles were obtained by keeping the contact strip/roll heat transfer coefficient constant equal to 10000 W/m²K and changing the inlet velocity value from 1 to 20 mm/s. The strip exit tension was kept constant equal to 20000 N for all the cases. It is found that when the inlet velocity is 1 mm/s, the stress free region above the solidus temperature is very small and ends near to the inlet. Stress intensity (i.e., the magnitude of Von-Mises stresses) is highest at the region near to inlet and in contact with the rolls. This is because after complete solidification, the roll force is transmitted to the strip which produces high compressive stresses in that region. The thermal gradients within the strip also contribute to these compressive stresses. The roll force is also high for lower inlet velocity as can be seen in Fig. 2. The stress intensity gradually decreases and then rises due to the effect of strip exit tension near the kissing point. When the inlet velocity is 20 mm/s, the stress free region extends more than halfway to the kissing point. This is because at a higher inlet velocity, the length of solidification (i.e., the solidification interval) is larger. At a higher inlet velocity, the magnitude of roll force transmitted to the strip is very low as can be seen in Fig. 2. Therefore, the magnitude of stress intensity is low in the region in contact with the roll.

Figure 7 shows the variation of Von-Mises equivalent stress...
stresses across the through thickness of strip at strip exit point for different inlet velocities and at constant contact strip/roll heat transfer coefficient of 10000 W/m² K. Von-Mises stress profile indicates that the stress intensity is much higher at the strip surface at strip exit point compared to the central region of the strip. The value of the stresses at the surface of the strip changes from 98 to 120 MPa when the inlet velocity is changed from 1 to 20 mm/s.

### 3.2.3 Effect of contact strip/roll heat transfer coefficient

Contact strip/roll heat transfer coefficient also affects the resultant stress profile considerably. Von-Mises equivalent stress profiles were obtained by keeping the inlet velocity constant equal to 10 mm/s and changing the contact strip/roll heat transfer coefficient value from 3000 to 15000 W/m² K. The strip exit tension was kept constant equal to 20000 N for all the cases.

When contact strip/roll heat transfer coefficient is 3000 W/m² K, the stress free region where the temperature is above the solidus temperature is very large and extends more than halfway through the kissing point. Stress intensity (i.e. the magnitude of Von-Mises stresses) is highest at the surface of the strip in contact with the rolls at the strip exit point. When the contact strip/roll heat transfer coefficient is 15000 W/m² K, the stress free region is smaller compared to the case of 3000 W/m² K and extends less than the halfway through the kissing point. The magnitude of Von-Mises stress at the strip exit point in contact with the rolls varied from 114 to 203 MPa when the contact strip/roll heat transfer coefficient is changed from 15000 to 3000 W/m² K. This indicates that at a lower value of heat transfer coefficient, the magnitude of stress intensity is higher at the surface of the exiting strip and so the strip is more prone to failure at a lower heat transfer coefficient than at a higher heat transfer coefficient.

Figure 8 shows the variation of Von-Mises equivalent stresses across the through thickness of strip at strip exit point for different contact strip/roll heat transfer coefficients and at constant inlet velocity of 10 mm/s. Once again, the Von-Mises stress profile indicates that the stress intensity is much higher at the strip surface at strip exit point compared to the central region of the strip. The value of the stresses at the surface of the strip changes from 193 to 114 MPa as the contact strip/roll heat transfer coefficient is changed from 3000 to 15000 W/m² K. The stress distribution for higher contact strip/roll heat transfer coefficient of 7000, 10000, and 15000 W/m² K is nearly the same, which indicates that at a higher contact heat transfer coefficient the stress distribution at the strip exit point does not change much with change in contact strip/roll heat transfer coefficient.

### 3.3 Stress analysis of rolls

Figure 9 shows the σ_y stress profile within the roll for inlet velocity of 10 mm/s and contact strip/roll heat transfer coefficient of 10000 W/m² K. Very high magnitude compressive stresses are developed along the surface of the rolls in contact with the strip. As the heat gets transferred from the strips to the rolls, the por-
tion of the rolls in contact with the strip tends to expand but is constrained by the solidified shell of the strip. Hence the surface of the rolls is subjected to a high compressive load. The compressive stresses developed are due to the combined effect of large thermal gradients across the through thickness of strips and also due to the compressive load applied on the rolls. The inner surface of the rolls is cooled by recirculating water, and so the magnitude of stresses on the inner surface are close to zero because there is a small temperature difference between the inner surface and the reference temperature. So, along the through thickness of the roll, the stresses change from compressive stresses of very high magnitude to stresses close to zero value. Stresses vary from $-292$ MPa at the outer surface to $-1.14$ MPa at the inner surface of the roll. As the roll looses contact with the strip, the magnitude of the compressive stresses at the rolls surface reduces and subsequently, stresses are close to zero when the rolls attain uniform temperature distribution close to the reference temperature of water ($313$ K). The stresses then attain a constant value, before the part of the roll comes in contact with the melt. In one cycle of rotation, the roll undergoes a stress cycle comprising of very high compressive stresses to nearly zero stresses. This is also known as the thermal fatigue cycle. The roll life is dependent upon the number of cycles that it can undergo before failure.

### 3.3.1 Effect of inlet velocity

Inlet velocity has a considerable effect on the stress distribution within the rolls. Stress profiles within the roll were obtained by keeping the contact strip/roll heat transfer coefficient constant equal to $10000$ W/m$^2$K and varying the inlet velocity value from 1 to 20 mm/s. For the case of lower inlet velocity of 1 mm/s, the maximum compressive stress developed within the roll region in contact with the strip is $-359$ MPa compared to $-321$ MPa for higher inlet velocity of 20 mm/s. For a lower inlet velocity, the solidification gets completed well before the exit point of the strip, hence rolling is quite significant in this case. At a constant value of contact strip/roll heat transfer coefficient $10000$ W/m$^2$K, the roll force increases with decrease in inlet velocity as shown in Fig. 2. These roll forces as well as the thermal gradients within the roll produce compressive stresses within the rolls. The magnitude of compressive stresses lowers as the roll loose contact with the strip. For inlet velocity of 1 mm/s the temperature distribution within the roll is quite uniform and so the stress distribution is also very uniform and stresses attain a constant value before the roll once again comes in contact with the melt. For inlet velocity of 20 mm/s the temperature distribution becomes uniform much away from the strip exit point and so there is a variation in stresses for a large part of the roll.

Figure 10 shows the variation of stresses along the roll surface for different inlet velocities and at constant contact strip/roll heat transfer coefficient of $10000$ W/m$^2$K.

### 3.3.2 Effect of contact strip/roll heat transfer coefficient

Contact strip/roll heat transfer coefficient affects the stress distribution within the roll region in contact with the strip. Stress profiles within the roll were obtained by keeping the inlet velocity constant equal to 10 mm/s and varying the contact strip/roll heat transfer coefficient from $3000$ to $15000$ W/m$^2$K. For the case of higher contact strip/roll heat transfer coefficient of $15000$ W/m$^2$K, the maximum compressive stress developed within the roll region in contact with the strip is $-308$ MPa compared to $-247$ MPa for lower contact heat transfer coefficient of $3000$ W/m$^2$K. This is because, the heat transfer per unit area is higher for a higher heat transfer coefficient and so the thermal gradient within the roll through thickness is very high, which produces higher compressive stresses. Moreover, the reaction roll force value is more for a higher heat transfer coefficient at a constant inlet velocity, as can be seen in Fig. 2. The stress distribution within the roll region not in contact with the strip does not change appreciably with change in inlet velocity. The stress distribution on the roll surface not in contact with the strip is not significantly affected by the change in inlet velocity.

Figure 11 shows the variation of $\sigma_y$ stresses along the roll surface for different contact strip/roll heat transfer coefficients.
surface for different contact strip/roll heat transfer coefficients and at constant inlet velocity equal to 10 mm/s. As the roll comes into contact with the melt, compressive stresses start developing on the surface and the magnitude of these stresses starts increasing as solidification within progresses from inlet. For a case of higher contact strip/roll heat transfer coefficient like 10000 and 15000 W/m²K, the maxima of compressive stress corresponds to the middle region of the roll in contact with the strip, whereas for lower heat transfer coefficient like 3000 W/m²K, the maxima of compressive stress corresponds to the roll region very near to the strip exit point. This is because for a higher heat transfer coefficient, the solidification takes place very early in the process and the temperature difference between the strip and the rolls lowers and so the thermal gradients between the roll and the strip surface decreases and the magnitude of the compressive stresses start decreasing. Whereas, in case of lower heat transfer coefficient, the solidification takes place near to the exit point of the strip and there is still a high temperature difference between the roll surface and the strip surface and so the magnitude of the compressive stresses continue to rise until, the rolls loose contact with the strip. The stress distribution on the surface of nodes not in contact with the strip does not change appreciably with change in strip/roll heat transfer coefficient as can be seen in Fig. 11.

4. Conclusions

The present study indicates that rolling force is an important parameter to be studied as it decides the amount of work hardening on the strip and also gives the indication of how much amount of energy is required for the rolling process. The variation of rolling force was in the range from 10³ to 10⁶ N for different operating conditions, taking the width of the strip to be equal to 1 m. It was found that Von-Mises equivalent stresses are highest at the surface of the strip in contact with the rolls at the strip exit point. This indicates that the surface of the strip is more prone to cracking than the center of the strip. The Von-Mises equivalent stresses at the strip exit point increases by increasing the inlet velocity and decreasing contact strip/roll heat transfer coefficient.

Stress distribution within a strip is not a strong function of change of heat transfer coefficient at a higher value of contact strip/roll heat transfer coefficient and similarly it is not a strong function of change of inlet velocity at higher inlet velocities. In one cycle of rotation, the roll undergoes a stress cycle comprising of compressive stresses. The compressive stresses are generated in the roll region in contact with the strip due to the combined effect of roll force and high thermal gradients within the rolls. Stress distribution within the roll region not in contact with the strip does not depend appreciably on variation of contact strip/roll heat transfer coefficient and inlet velocity.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Greek Symbols</th>
<th>Description</th>
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<tr>
<td>$\alpha$</td>
<td>Thermal coefficient of expansion (K⁻¹)</td>
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<td>$\alpha_x$</td>
<td>Thermal coefficient of expansion in x direction (K⁻¹)</td>
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<tr>
<td>$\alpha_y$</td>
<td>Thermal coefficient of expansion in y-direction (K⁻¹)</td>
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<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker Delta</td>
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<td>$\epsilon_{ij}$</td>
<td>Summation of stresses in x, y, and z direction</td>
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<table>
<thead>
<tr>
<th>Symbols</th>
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<td>$a$</td>
<td>Strain rate sensitivity of hardening or softening</td>
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<td>$A$</td>
<td>Pre-exponential factor (s⁻¹)</td>
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<tr>
<td>$n$</td>
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<td>Cartesian coordinate in vertical direction</td>
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<td>$W$</td>
<td>Width of the strip (m)</td>
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\( \varepsilon_{ij} \) Strain tensor
\( \varepsilon_{ij}^e \) Elastic part of strain tensor
\( \varepsilon_{ij}^p \) Plastic part of strain tensor
\( \varepsilon_{ij}^T \) Thermal part of strain tensor
\( \dot{\varepsilon}_{ij}^p \) Plastic strain rate (s\(^{-1}\))
\( \xi \) Multiplier of stress
\( \kappa \) Term in eq. (10)
\( \lambda \) Lamé’s constant
\( v \) Poisson’s ratio
\( \sigma_1, \sigma_2, \sigma_3 \) Principal stresses (Pa)
\( \sigma_e \) Von-mises equivalent stress (Pa)
\( \sigma_x \) Stress in X-direction (Pa)
\( \sigma_y \) Stress in Y-direction (Pa)
\( \sigma_{ij} \) Stress tensor (Pa)
\( \sigma_{ij}^d \) Deviatoric part of stress tensor (Pa)
\( \sigma_{\text{yield}} \) Mean yield stress (Pa)

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