Flatting the Solidification Front by Varying the Wall Thickness of the Mould in Directional Solidification Technology

Yuanyuan Lian¹,²,*¹, Dichen Li³,⁴ and Kai Zhang⁵,*²

¹School of Aerospace, Xi’an Jiaotong University, Xi’an 710049, PR China
²State Key Laboratory for Strength and Vibration of Mechanical Structures, Xi’an Jiaotong University, Xi’an 710049, PR China
³State Key Laboratory for Manufacturing System Engineering, Xi’an Jiaotong University, Xi’an 710049, PR China
⁴School of Mechanical Engineering, Xi’an Jiaotong University, Xi’an 710049, PR China
⁵School of Mathematics and Statistics, Xi’an Jiaotong University, Xi’an 710049, PR China

A non-planar solidification front can cause many defects in the directional solidification process. The irregular shape of the cross section of the cast leads to a non-uniform heat distribution and hence causes a non-planar solidification front. This paper provides a method to decrease the circumferential temperature gradient in the directional solidification process by varying the wall thickness of the mould. The relationship between the wall thickness and the control parameter has been determined. An optimization algorithm for obtaining the control parameter and the corresponding wall thickness of mould is presented with three simulation examples provided to verify the superiority of this method. According to the results the circumferential temperature gradients can be reduced by 68.7%, 72.0% and 88.2% respectively. Moreover, the transverse temperature gradients are reduced by 57.3%, 60.8% and 89.3% accordingly. The solidification front with a varying thickness mould is thus flattened compared with the un-optimized one. [doi:10.2320/matertrans.M2016157]

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1. Introduction

In aviation industry, the demand for single crystal turbine blades increases because of the excellent performance¹. However, many manufacturing defects of the single crystal blade have not been eliminated, such as freckles², stray grains³ and high dislocation levels⁴ etc. One of the major factors in causing defects is the nonplanar solidification front⁵-⁶. Much research has been carried out for flattening the solidification front. The solid-liquid interface position in furnace⁷, the size of the cast and the alloy characteristics⁸ are the main factors in influencing the shape of solidification front. The solid-liquid interface position is determined by the axial temperature gradient and withdrawal rate etc.⁹. Monastyrskiy⁹ provided a numerical optimization method on withdrawal rate in directional solidification process for obtaining a planar solidification front. Trivedi et al.¹⁰ found that the solidification front is more planar as the diameter of the cast is smaller. El-Mahallawy and Farag¹¹ discovered that the solidification front is more planar when the ratio of thermal conductivity of the alloy to that of the mold is larger. Adorno and Brown¹² also demonstrated that the solidification front can be flattened by decreasing the thermal conductivity of the mould.

Except the studies listed above, some other methods were proposed to flatten the solidification front. Jasinski and Witt¹³ and Voiz et al.¹⁴ showed that localized heating toward the solid-liquid interface can flatten the solidification front during Bridgman process. Zhang et al.¹⁵ provided a bell-curve thermal profile of the furnace for improving the solid-liquid interface shape during single crystal growth of a bar. Tortorelli et al.¹⁶ provided a general method for deriving direct and adjoint sensitivities for nonlinear parabolic systems. They applied this method to optimize the solidification process in a Bridgman furnace. The solidification front is flattened by a specified temperature distribution obtained by controlling the temperature on the furnace wall. Ebrahimi et al.¹⁷ optimized investment casting, such as the rigging systems and the mold wrap by finite element solidification heat transfer analysis and design sensitivity analysis. The solidification pattern can be controlled by the thickness of the mold wrap.

From the research above, it can be inferred that reducing the transverse temperature gradient is the essential way to flatten the solidification front. Pfeiffer and Mühlberg¹⁸ studied the way that reduces transverse temperature gradient. They indicated that a small diameter of the cast results in a small transverse temperature gradient. Haldenwang and Guérin¹⁹ investigated the mechanisms resulting in the transverse temperature gradient in directional solidification as well. They employed rod samples and studied the influence of the furnace aspect ratio and the ratio of the conductivity of the mould to that of the sample etc. in transverse temperature gradient. However, the cast with irregular shape was not considered by above studies. For a cast with irregular cross sections, the transverse temperature gradient is complex in the solidification front, and it can be divided into radial temperature gradient and circumferential temperature gradient (cf. Fig. 1 in Ref. 20)). Lian et al.²⁰ provided a method of varying the thickness of the mould for the reduction of the circumferential temperature gradient. The results showed that the solidification front is more planar by reducing the circumferential temperature gradient. Moreover, this method can be further optimized.

This paper provides an optimization method based on the study of Lian et al.²⁰ for decreasing the circumferential temperature gradient and hence flattening the solidification front.
The most appropriate wall thickness of the mould is obtained by a new optimization process. Section 2 introduces the control parameter of our optimization method and the algorithm for obtaining the wall thickness. In Section 3, three simulation examples are presented and discussed. And this paper is concluded in Section 4.

2. Theory

For a cast with an irregular cross section, the heat dissipation along the circumferential direction is nonuniform in directional solidification (e.g. high rate solidification process). For instance, the blade possesses both concave and convex structure at one cross section. The heat dissipation at trailing edge is larger than at other parts due to the larger configuration factor. The nonuniform heat dissipation leads to circuferential temperature gradient and then nonplanar solidification front. The iso-thickness mould, which the wall thickness are equal at different positions, cannot reduce the circumferential temperature gradient. That is, the wall thickness varies with the positions. The temperature gradient along circumferential direction can be almost reduced by 40%. This method can be improved and the temperature gradient can be reduced further. An optimization algorithm is put forward in this section for obtaining the optimal wall thickness. Besides, three simulation examples are provided to verify the superiority.

2.1 Optimization on the wall thickness

In this subsection, a control parameter is determined first. The relationship between the control parameter and the wall thickness is deduced. Then an optimization algorithm is put forward for obtaining the optimal mould.

2.1.1 Control parameter of the wall thickness

The method by Lian et al.\textsuperscript{(20)} utilized varying thickness moulds to balance the heat dissipation along the circumferential direction. Thus the heat distribution at the solidification front is more uniform than that of the un-optimized one. A set of equations concerning the heat dissipation in solidification is more uniform than that of the un-optimized one. A set of equations concerning the heat dissipation in solidification factor. The nonuniform heat dissipation leads to circuferential temperature gradient and then nonplanar solidification front. The iso-thickness mould, which the wall thicknesses at different positions, cannot reduce the circumferential temperature gradient. That is, the wall thickness varies with the positions. The temperature gradient along circumferential direction can be almost reduced by 40%. This method can be improved and the temperature gradient can be reduced further. An optimization algorithm is put forward in this section for obtaining the optimal wall thickness. Besides, three simulation examples are provided to verify the superiority.

\[ q_r(P) = \lambda \left( \frac{T_2(P) - T_1(P)}{\delta(P)} \right) \]  

(1)

\[ q_r(P) = \frac{C_p(T_1^2(P) - T_2^2)}{1 - \varepsilon_1} \frac{1}{X(P)} \]  

(2)

\[ q_r(P) = q_c(P), \forall P \in \Gamma \]  

(3)

and

\[ q_r(P_1) = q_r(P_2), \forall P_1, P_2 \in \Gamma \]  

(4)

In eq. (1)–(4), \( q_r(P) \) and \( q_c(P) \) denote the heat flux of the thermal conduction and the heat flux of the radiation at an arbitrary point \( P \) respectively. \( T_2(P), T_1(P) \) and \( T_0 \) are the temperatures of the external surface \( \Gamma \) of the cast, the external surface \( \Gamma' \) of the mould and the cooling ring respectively. \( \delta(P) \) is the thickness at \( P \) and \( X(P) \) is the configuration factor at \( P \). \( \lambda \) is the thermal conductivity of the mould and \( \varepsilon_1 \) is the emissivity of the mould surface. For more details, see literature\textsuperscript{(20)}

These physical and geometrical parameters are fixed during the solidification process and need to be set beforehand in simulation except the parameter \( T_2 \) which is the temperature of the external surface of the cast and decreases in time. In order to calculate the wall thickness by above equations, we need to choose a fixed value of \( T_2 \), which we call \( T_{20} \). That is, the parameter \( T_{20} \) is the nominal temperature of the external surface of the cast and doesn’t need to be set in simulation. In actual manufacturing, the temperature of the external surface of the cast is time-dependent. The ideal wall thickness of the mould should change with the temperature of the external surface of the cast. However, the wall thickness of the mould cannot change during solidification process. Therefore, it is important to find an optimal mould which has the best effectiveness in flattening the solidification front. That is, the transverse temperature gradient during solidification, i.e., at the melting point temperature is reduced to the minimum by the optimal mould. However, \( T_{20} \) cannot be set as the melting point temperature for our purpose because the temperatures at the solidification front are affected by the temperatures of the molten metal and the temperatures at any time are influenced by the temperatures at previous time. It implies that \( T_{20} \) should be a smaller value. This phenomenon is illustrated in Ref. \textsuperscript{20} (cf. Fig. 14(b)). Thus, an appropriate \( T_{20} \) is needed. The optimal mould can be obtained by the equations with an appropriate \( T_{20} \). We call \( T_{20} \) the control parameter and it is the only control parameter of the wall thickness of the mould.

The heat distribution on a cross section \( C \) can be uniformized by eliminating the circumferential temperature gradient on \( C \) in directional solidification. Based on eq. (1)–(4) in Ref. 20, we can deduce the following formula

\[ T_{20} = a \sqrt{\frac{a - 1}{\delta \Delta}} + b \sqrt{\frac{a - 1}{\delta \Delta}} \]  

(5)

where \( a = \sqrt{X(P_0)/X(P_1)}, \Delta = \delta(P_1) - \delta(P_0), a = \sqrt{X(P_0)/X(P_1)} \) and \( b = \delta \delta(P_0) \). The points \( P_0 \) and \( P_1 \) lie on the outer contour of the cast (i.e., the boundary of \( C \)), where \( P_0 \) denotes the point having the minimum configuration factor and \( P_1 \) is an arbitrary point. \( X(P_0) \) and \( X(P_1) \) are the configuration factors at \( P_0 \) and \( P_1 \) respectively. \( \delta(P_0) \) and \( \delta(P_1) \) are the wall thicknesses at \( P_0 \) and \( P_1 \) respectively. Because \( P_0 \) has the minimum configuration factor, \( \delta(P_0) \) is the minimum thickness of the varying thickness mould. We abbreviate \( \delta(P_0) \) as \( \delta_0 \), and define it as the base wall thickness. \( \lambda \) is the thermal conductivity of the mould. And \( C_b \, (=5.67 \times 10^{-8} \, \text{W/(m}^2 \cdot \text{K}^4)) \) is the black body radiation coefficient. When \( P_1 \) is the point with maximum configuration factor, the parameter \( \Delta \) means the thickness difference of the varying thickness mould, i.e., \( \Delta = \max_{P_0 \leq P \leq P_1} \delta(P) - \min_{P \leq P_0} \delta(P) \). In the rest of this paper, \( \Delta \) always denotes the thickness difference of the varying thickness mould.

We drop the parameter \( T_{20} \) which represents the temperature of the cooling ring, during the formula derivation. It is
because $T_0$ impacts little for the wall thickness$^{20}$.

Equation (5) shows clearly the relationship between the wall thickness and the control parameter $T_{20}$. And the wall thickness can be obtained directly by eq. (5) instead of using the four equations in Ref. 20). Equation (5) implies that the thickness difference $\Delta$ decreases in $T_{20}$. For balancing the heat dissipation of a thin cast, parameter $T_{20}$ should be small. According to eq. (5), we can also obtain that the rate of change of $\Delta$ decreases in $T_{20}$ as well. It indicates that $\Delta$ is more sensitive for smaller $T_{20}$.

From eq. (5), $\lambda$, $\delta_0$ and $X(P_1)/X(P_0)$ are the main factors in influencing the thickness difference of the mould. Figure 1 shows the variation of $\Delta$ with $T_{20}$, when the thermal conductivity $\lambda$, the base wall thickness $\delta_0$ and the ratio $X(P_1)/X(P_0)$ of the maximum configuration factor to the minimum configuration factor change. The basic parameters are $\lambda = 2$ W/m/K, $\delta_0 = 9$ mm and $X(P_1)/X(P_0) = 1.5158$, i.e., two of the parameters are remained unchanged for the computation, which are the same as the parameters in Ref. 20). It can be seen that $\Delta$ increases in $\lambda$, $\delta_0$ and $X(P_1)/X(P_0)$ when $T_{20}$ is fixed. Moreover, the impact of $\lambda$ and $X(P_1)/X(P_0)$ are larger than that of $\delta_0$. These observations can be used to guide the design of the furnace and the choice of the material of the mould.

2.1.2 Optimization algorithm of the wall thickness

From Ref. 20), the circumferential temperature gradient can be almost reduced by 40%. However, Fig. 8 (b) Ref. 20) illustrates that the heat dissipation at trailing edge becomes the largest after the solidification begins and the temperature becomes the lowest. This implies that to reduce the circumferential temperature gradient further, the wall thickness at trailing edge should be larger, i.e., the thickness difference $\Delta$ should be larger. Equation (5) indicates that $T_{20}$ should be smaller. Hence, it can be expected that the circumferential temperature gradient reduces much with a proper $T_{20}$ which is smaller than 1573 K (1300°C) as shown in Ref. 20).

Given a $T_{20}$, the wall thickness can be calculated by eq. (5) with the corresponding parameters and the mould contour can be obtained accordingly. Next, we build the three-dimensional solidification simulation model on the basis of the outer contour of the cast and mould contour to compute temperature distribution changing with time. We can use the circumferential temperature difference $DT_c$ to describe the circumferential temperature gradient; and $DT'_c = \max(\max_{P \in C} T_2(P) - \min_{P \in C} T_2(P))$, where $t$ is the time, $C$ is the cross section and $T_2(P)$ is the temperature of the cast at $P$. The temperature difference ratio $DT'_c$ is the ratio of the temperature difference with the varying thickness mould to that of the iso-thickness mould. We need to find an appropriate $T_{20}$ such that $DT'_c$ is the lowest.

Figure 2 shows the flow chart for obtaining the optimal $T_{20}$ and the corresponding varying thickness mould. First, we choose three initial values $T_{20}^1$, $T_{20}^2$, $T_{20}^3$ to construct the varying thickness mould by eq. (5) and simulate the solidification process respectively. The initial values are set to build a base interval to search the optimal $T_{20}$ and $T_{20}'$ should be large in this algorithm. The solidus temperature can be chosen for $T_{20}'$. $U_1$ and $U_2$ are the parameters to shorten available interval for $T_{20}$. That is, $U_1$ and $U_2$ may decrease after an iterative cycle and $U_1$ is always bigger than the optimal $T_{20}$. The initial search direction is along the direction that $T_{20}'$ decreases. $C_1$ and $C_2$ are the tolerances in stopping criterion. These values don’t need to be very small. Here, we set $C_1 = 30$ K (°C) and $C_2 = 1$ K (°C). $L'$ is the step length for the $i$–th iterative cycle. $L'$ can be large at the beginning and should be small for small $T_{20}$ due to the sensitivity of thickness difference $\Delta$ on $T_{20}$. The flow chart provides an algorithm to obtain the optimal $T_{20}$ in strict logic.

![Figure 1](image_url) Flattening the Solidification Front by Varying the Wall Thickness of the Mould

![Figure 2](image_url)
2.2 Simulation

In this subsection, we provide three numerical examples to verify the superiority of our optimization method. We employ the turbine blades as the casts in simulation. The superalloy CMSX-2 is adopted for the blade. The model and the simulation process are similar to that of Ref. 20. Roughly speaking, the simulation process is as follows. First, we use the optimization procedure provided above to construct the mould contour. And then we employ UG software to build the three-dimensional model. Afterwards, we simulate the solidification process by ProCAST. The axial temperature gradient is 30 K/cm in directional solidification process. The temperature in hot zone is 1773 K (1500 °C) and the temperature of cooling ring is set to 308 K (35 °C) for all simulations. The key parameters for simulation are illustrated in Table 1. The details for the simulation process are shown in Ref. 20.

After simulation, we use the circumferential temperature difference $DT_c$ and the ratio $DT_c/T_{20}$ to verify the superiority of our optimization method. As done in Ref. 20, we choose four typical points $P_t$, $P_s$, $P_l$, and $P_p$ at trailing edge, suction surface, leading edge, and pressure surface respectively to calculate the circumferential temperature difference $DT_c$. These four points are on the boundary of one cross section. The temperatures of these four points are the most different along the circumferential direction. Hence, $DT_c = \max_{P \in S_T} \{T_{20} - T(P)\}$, where $S = \{P_t, P_s, P_l, P_p\}$. Besides, we also use the transverse temperature difference $DT_t$ to evaluate the effectiveness. It is defined as $DT_t = \max_{P \in C_T} \{T_{20} - T(P)\}$, where $C$ denotes the cross section we have chosen. Similarly, we can define the temperature difference ratio along the transverse direction $DT_t/T_{20}$ as above. Note that $S \subset \partial C \subset C$. Hence, $DT_c$ is always smaller than $DT_t$.

### 3. Results and Discussion

The solidification processes were simulated by ProCAST according to the program presented in last section and the optimal $T_{20}$ were obtained for the three examples. In this section, the results are provided and discussed to verify the superiority of our optimization method and suggestions were offered for actual manufacturing. The improvement of the calculation program is presented as well.

#### 3.1 Superiority of the optimization method

In this subsection, the superiority of our method is verified by the three examples. Figure 3 presents the schematic diagrams of the typical points $P_t$, $P_s$, $P_l$, and $P_p$ in blade 1 and blade 2 respectively. These four points lie on one cross section which is 5 mm far from the top of the blade for each example. The temperatures at these points are used to calculate the temperature difference $DT_c$. For the blade used in example 3, the point $P_t$ is not close to $P_0$. Instead, $P_t$ lies in the middle of the suction surface where is far away from the trailing edge and leading edge. Because the blade is thin and $P_0$ is near the leading edge, the heat dissipates quickly from $P_0$ to the leading edge. The heat dissipation at $P_t$ is less influenced by the two edges and the temperature here decreases the most slowly.

Figure 4, Fig. 5 and Fig. 6 show the temperature-time

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<th>Table 1 The key parameters for simulation.</th>
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<td>Parameters</td>
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curves with the iso-thickness mould and the varying thickness mould for the three examples. The varying thickness mould is obtained with optimal $T_{20}$ and this is understood throughout this subsection. The figures show that the circumferential temperature gradients are significantly reduced. For an iso-thickness mould, the temperatures at trailing and leading edges decrease faster than at other parts due to the large configuration factors. By employing the varying thickness mould, the heat dissipation is balanced and temperatures of different parts diverge slightly. Hence, the circumferential temperature gradient is small during solidification. In addition, it can be seen that, for a varying thickness mould, the temperature at trailing edge is larger than at suction surface before the start of solidification. It is because the wall thickness at trailing edge is larger than at suction surface. From Fig. 4(b) and Fig. 5(b), the temperature at leading edge is obviously lower than at other parts after a period of heat dissipation. It means that the wall thickness at leading edge is not large enough to balance the heat dissipation along the circumferential direction. Therefore, the circumferential temperature gradient may be further reduced by enlarging the thickness at the leading edge.

The circumferential temperature difference $D_{T_c}$ and the circumferential temperature difference ratio $D_{T_c}'$ can be calculated by using the method put forward in Section 2. Table 2 illustrates the values for each example. Here, $D_{T_c}$ denotes the circumferential temperature difference with the iso-thickness mould. As shown in Table 2, the temperature gradient along the circumferential direction is almost reduced by 70% for example 1. The ratio $D_{T_c}'$ is 31.3% and is almost half of the one (60.4%) calculated in Ref. 20) although the physical and geometrical parameters are the same. It clearly demonstrates the effectiveness of our optimization method. For example 2, the circumferential temperature gradient decreases by more than 70%. It indicates that this method is applicable for different base wall thicknesses. The circumferential temperature difference for example 3 with the iso-thickness mould is very large [699.7 K (426.7°C)]. It is because the blade near the

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Fig. 3  Schematic diagram of choosing the four typical points and two longitudinal sections on the blade 1 (a) and blade 2 (b).

Fig. 4  Temperature-time curves with the iso-thickness mould (a) and the varying thickness mould with optimal $T_{20}$ (b) for example 1.

Fig. 5  Temperature-time curves with the iso-thickness mould (a) and the varying thickness mould with optimal $T_{20}$ (b) for example 2.
trailing edge is thin and heat dissipates quickly there. This leads to the large difference of heat radiation along the circumferential direction and hence large temperature gradient. In this case, our method is more effective. Indeed, the temperature gradient along circumferential direction is almost reduced by 90%.

According to the stress equilibrium equations in a temperature field\(^{21}\), the stress is caused by the temperature gradient. A large circumferential temperature gradient leads to large stress and thus results in hot crack. By employing our method, the stress and hence the fraction can be reduced during directional solidification.

Additionally, Table 2 shows that the optimal \(T_{20}\) and \(DT_c\) of example 1 is almost the same as that of example 2. It implies that the optimization is related closely to the shape of the blade. The ratio \(DT_f\) of example 1 is slightly more than that of example 2 because \(DT_c\) of example 2 is higher more than 20 K (°C). It means that, for iso-thickness moulds, the circumferential temperature gradient is larger if the wall thickness is thinner. Thus, our method is more effective for the mould with smaller base wall thickness. In addition, the optimal \(T_{20}\) for example 3 is quite small. It implies that the optimal wall thickness of the mould is larger for a thinner blade.

The heat dissipation along the circumferential direction is thus balanced. Moreover, the transverse temperature difference \(DT_f\) is also reduced. It is because the temperature gradient along the circumferential direction is positively related to the temperature gradient along the transverse direction. The values of \(DT_f\) and \(DT_r\) are also shown in Table 2. The transverse temperature gradients are reduced by 57.3%, 60.8% and 89.3% respectively for the three examples. The reduction of transverse temperature gradient is smaller than that of the circumferential temperature gradient except example 3. The \(DT_f\) of example 3 is rather large because the heat dissipates fast at the thin trailing and leading edges. The \(DT_r\) of example 3 is rather small because the thickness difference is quite large.

Table 2 also illustrates that the transverse temperature gradients are larger than the circumferential temperature gradients. For example 1 and example 2, the \(DT_f\) are about 39 K (°C) larger than \(DT_c\). And the \(DT_r\) are about 47 K (°C) larger than \(DT_c\). It can be inferred that the difference between transverse temperature gradient and circumferential temperature gradient is almost unchanged for the same blade. And the difference with the varying thickness mould is smaller than with the iso-thickness mould. The \(DT_f\) and \(DT_r\) are 4 K (°C) and 78 K (°C) larger than \(DT_c\) and \(DT_r\) respectively for example 3. It implies that the transverse temperature gradient reduces more for a thin blade. It means that our method is more effective for a thin blade.

Gasperino et al.\(^{4}\) stated that the solid-liquid interface coincides with the solidus isotherm. It can be inferred that smaller transverse temperature gradient means the more planar solidification front. Hence, the solidification front is flattened accordingly with the reduction of the transverse temperature gradient. Figure 7, Fig. 8 and Fig. 9 show the isotherms with the iso-thickness mould and the varying thickness mould at \(t = 40s\) for example 1, example 2 and example 3 respectively. It can be seen that the solidification front with the varying thickness mould is more planar than with the iso-thickness mould.

More precisely, we introduce the nonflatness \(h\) to measure the effectiveness of our method quantitatively. The nonflat-
ness $h$ is defined as the maximum of the height difference of each isotherm on one longitudinal section. And $h_1$ [resp. $\bar{h}_1$] and $h_2$ [resp. $\bar{h}_2$] correspond to $S_1$ and $S_2$ with the varying thickness [resp. iso-thickness] mould respectively. Table 3 shows that the nonflatness $h$ is reduced notably. Additionally, the nonflatness $h$ is related to the height of the simulation model. The flatness $h$ for example 2 is larger than that for example 1 due to the larger height of the simulation model. In addition, the ratio of the nonflatness for example 2 is smaller than that for example 1. It implies that our method is more appropriate for the large size blade. Besides, the nonflatness $h$ for $S_1$ is smaller than that of $S_2$ because the lateral dimension

<table>
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<tr>
<th>Example</th>
<th>$\bar{h}_1$ (mm)</th>
<th>$\bar{h}_1$ (mm)</th>
<th>$h_1/\bar{h}_1$</th>
<th>$\bar{h}_2$ (mm)</th>
<th>$\bar{h}_2$ (mm)</th>
<th>$h_2/\bar{h}_2$</th>
</tr>
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<tr>
<td>Example 1</td>
<td>14.5</td>
<td>5.9</td>
<td>40.7%</td>
<td>16.9</td>
<td>8.5</td>
<td>50.3%</td>
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<tr>
<td>Example 2</td>
<td>44.5</td>
<td>5.0</td>
<td>11.2%</td>
<td>46.2</td>
<td>14.4</td>
<td>31.2%</td>
</tr>
<tr>
<td>Example 3</td>
<td>40.8</td>
<td>6.5</td>
<td>15.9%</td>
<td>46.6</td>
<td>9.7</td>
<td>20.8%</td>
</tr>
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</table>

3.2 Further discussion

Figure 10 shows the mould contours with different $T_{20}$ for the three examples. According to Fig. 10, the wall thicknesses at the trailing edge and the suction surface are the largest and the smallest respectively. This is because the configuration factors of these two parts are the maximum and the minimum respectively. The thickness differences $\Delta$ with the optimal $T_{20}$
The thickness difference for example 1 is larger than that for example 2 although the same blade is employed. It indicates that the shape of the mould is influenced by parameters $\lambda$, $H$ and $\delta_0$. Smaller thermal conductivity of the mould leads to more planar solidification front\textsuperscript{11,12,18}. When choosing a material with small $\lambda$ for planar solidification front, the thickness difference is small correspondingly by means of our method (cf. Fig. 1(a)). A mould with small thickness difference is easy to be manufactured and the materials are conserved. It means that the solidification front can be flattened without increasing the difficulty of the manufacturing. In addition, large $H$ means small ratio of maximum configuration factor to minimum configuration factor\textsuperscript{20}. Combining with Fig. 1(c), it can be inferred that the thickness difference decreases in $H$. Hence, small $\lambda$ and large $H$ are suggested in actual manufacturing. Besides, small base wall thickness $\delta_0$ means the small thickness difference. Hence, the base wall thickness $\delta_0$ can be chosen as small as possible based on the premise that the mould meets the strength requirement.

As can be seen in Fig. 10 (b) and (c), the thickness difference $\Delta$ is larger when the blade is thinner. It is because the heat on trailing edge and leading edge dissipates fast for a thin blade. The mould with much larger $\Delta$ can balance the nonuniform heat dissipation. In order to manufacture blades with different scales, a large height $H$ of cooling ring of the furnace and a small thermal conductivity $\lambda$ of the mould are suggested.

The solid lines in Fig. 11 present the optimization process and the corresponding calculation results of each example. It can be seen that the temperature difference ratio $DT_c$ decreases considerably and the calculation steps are less than 10 for all examples. It implies that our method is effective without huge computational cost. As shown in Fig. 11(a) and Fig. 11(b), the optimization steps are similar for example 1 and example 2. It implies that the optimization process is closely related to the shape of the blade.

Figure 11(c) displays that $DT_c$ decreases slowly for large $T_{20}$ and fast for small $T_{20}$, which is rather different from the trend of example 1 and example 2. It indicates that the geometrical parameter, viz. blade contour is the principle factor for the optimization process. For a thin blade, the step length $L_i$ for larger $T_{20}$ can be set larger.

According to Fig. 4, Fig. 5 and Fig. 6, the solidification time with the varying thickness mould is more than with the iso-thickness mould. It is because the wall thickness of the varying thickness mould is larger than that of the iso-thickness mould. The dotted lines in Fig. 11 show the solidification time ratios $t_s$ for the three examples with different $T_{20}$. The ratio $t_s$ denotes the ratio of the solidification time with the varying thickness mould to that of the iso-thickness mould. From Fig. 11, the solidification time ratios with the optimal mould are 3.1, 2.5 and 18.2 respectively for the three examples. It means that the time efficiency decreases by applying the varying thickness mould.

Figure 11 presents both the effectiveness of the optimization method and the solidification time ratios for different $T_{20}$. As $T_{20}$ decreases, the thickness difference increases (cf. eq. (5) and Fig. 10). The transverse temperature gradient can be reduced significantly but the time efficiency is also lowered. In actual manufacturing, the time efficiency and the quality should be balanced. The $T_{20}$ for a varying thickness mould used in actual manufacturing can be chosen base on the figures similar to Fig. 11.

### 3.3 Improvement of the calculation program

The optimization method provided in this paper is on the basis of the study by Lian et al.\textsuperscript{20} Nevertheless, the calculation program such as the MATLAB program is improved. The improvements are stated as follows. The mould contour
is obtained by the MATLAB program and the coordinates of the blade contour. The discrete points of the blade contour can be generated directly by UG and exported as an iges file. The iges file can be saved as txt format and then exported to MATLAB without any processing. This improvement makes the method be applied conveniently for different blades. The system error is thus reduced and the computation time is decreased. Besides, we divide the MATLAB program into two subprograms. One is used to calculate the configuration factors for all points on the blade contour. The other is used to calculate the varying wall thickness. Hence, the MATLAB programs are clearer and more concise.

Additionally, it must be noted that the smoothing program changes with the blade contours. The wall thicknesses change dramatically at positions where the configuration factors change dramatically. This leads to the roughness of the mould contour. Hence, these parts need to be smoothed. For different blades, the positions to be smoothed are different.

Here, we suggest three smoothing methods: cubic spline data interpolation, polynomial curve fitting in least square sense and weighted average method. The weighted average method is to calculate the thickness at some point by taking the weighted average of the thicknesses around. The first method can eliminate the small zigzags and make the mould contour smooth. But it may lead to unreasonable deformation (cf. Fig. 12(a) with \( H = 10 \) mm in Ref. 20)). The second method can eliminate some large zigzags but may also lead to unreasonable deformation. The shape of the mould contour can be maintained by the third method. But this processed contour may be not smooth enough. The smoothing method and the part to be smoothed change with the blades. The mould contour varies dramatically when \( T_{20} \) is very small. The smoothing on the mould contour may not be accomplished only by MATLAB program. The mould contour should be further smoothed in UG and then used to build the entity model. The smoothing by UG is usually not needed.

4. Conclusions

In this paper, an optimization method for flattening the solidification front in directional solidification, which is based on varying the wall thickness of the mould, has been put forward. The formula for the relationship between the wall thickness and the control parameter \( T_{20} \) has been deduced. In addition, an optimization algorithm for obtaining the optimal varying thickness mould has been presented. Besides, three simulation examples applied the optimization method have been presented to verify the superiority of our method. The circumferential temperature gradient can be reduced by 68.7\%, 72.0\% and 88.2\% respectively for the three examples. Furthermore, the transverse temperature gradients are also reduced by 57.3\%, 60.8\% and 89.3\% respectively. Therefore, the solidification front is flattened for directional solidification. The defects may be reduced by applying the optimization method. In addition, this method is more appropriate for the blade with large size or sharp trailing (leading) edges. Besides, a large height of cooling ring of the furnace and a small thermal conductivity of the mould are suggested in the actual manufacturing.

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