An Investigation of Reflection Coefficients of the T(0,1) Mode Guided Waves at Axisymmetric Defects and Inverse Problem Analyses for Estimations of Defect Shapes

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The frequency dependences of the reflection coefficients at gradual step-down axisymmetric defects were experimentally evaluated for the investigation of the reflection phenomena of the T(0,1) mode guided waves. Different frequency dependences were observed depending on the axial profile of the defect. A mathematical model using the characteristic acoustic impedance for calculating the reflection coefficients at axisymmetric defects was introduced. The reflection coefficients for the gradual step-down axisymmetric defects were confirmed to be in good agreement with the calculation results. By utilizing the mathematical model, a reflection mechanism was explained precisely and was verified. An inverse problem analysis for estimating the shape of an axisymmetric defect was also proposed based on the experimental reflection coefficients as a function of frequency together with the calculation model. Experimental verification of the proposed estimation method was performed using circular concave axisymmetric defects inside pipe specimens. The estimation results and accuracy were discussed in terms of the relation between the frequency range and the axial length of the defects. [doi:10.2320/matertrans.M2014331]

Keywords: nondestructive evaluation, ultrasonic guided wave, T(0,1) mode, defect detection, defect sizing, inverse problem

1. Introduction

Guided wave inspections were very efficient method for indicating the axial locations of defects in piping. However it is ill-suited to accurate estimation of defect sizing comparing to the conventional ultrasonic testing (UT). In the early stage of 1970s, the experimental outcomes of defect detections were presented. However the evaluations of the defect detections were not yet quantitative. To improve the quantitativeness of the guided wave method, several basic investigations for the scattering phenomena at defects have been carried out. The effects of circumferential extent, axial extent, and depth for the defects have been well summarized experimentally numerically and theoretically for the longitudinal and flexural mode guided waves. Those effects for the torsional mode guided waves have been well summarized elsewhere either.

However, in the investigations described above, the defects had flat-bottomed geometries, in which the depths changed abruptly at the beginning and ending parts of the defects. This type of defect is very useful for the fundamental investigations. However, actual corrosive defects normally have distributions that change gradually in thickness. Therefore, studies on the scattering phenomena of the guided waves at defects that gradually change thickness are an important and essential issue in this research field. The reflection coefficients of several defects (tapered step-down, tapered step-up, tapered defect, and V-notch) were precisely investigated. The numerical analyses have also been carried out for the investigations. The boundary elementary method (BEM) has been applied to the investigation of propagation phenomena at gradually decreasing sections (half-elliptic defects) simulating corroded surfaces. The finite elementary method (FEM) has also been applied to this problem. Another approach to understand the reflection phenomena has been carried out. Two calculation models of the reflection coefficients at gradually changing defects have been proposed. These two methods are very useful for calculating the reflection coefficients of axisymmetric defects either, which must be useful for applications to solve inverse problems of the axisymmetric defects. However, the methods have not yet been adequately applied to the inverse problems for axisymmetric defects.

This paper shows experimental verifications of the reflection coefficients of the T(0,1) mode guided waves at gradual step-down axisymmetric defects together with their comparison with the calculation model. The calculation model was first explained and was applied to a simple example for understanding the reflection phenomena. A mechanism forming the waveform depending on the axial profile of the defect was explained with the calculation model. Secondly, the waveforms detected in three different tapered step-down axisymmetric defects for different frequencies were shown and discussed with the calculation results. The calculation results were confirmed to have coincided excellently with the experimental results. A novel approach to an inverse problem for estimating the shape of an axisymmetric defect was proposed using the calculation model and was verified. The accuracy of the estimations was discussed.

2. Calculation Model of the Reflection Coefficient at an Axisymmetric Defect

In the general case, an ultrasonic wave reflects at an interface between two half spaces having different specific acoustic impedances \( Z_n = \rho_n v_n \), where \( \rho_n \) and \( v_n \) are the material density and velocity of the ultrasonic wave (\( \eta \) is a countable index indicating the medium), respectively. The reflection and transmission coefficients, \( r_{12} \) and \( t_{12} \), from medium 1 to medium 2 are given as follows:

\[
\frac{Z_2 - Z_1}{Z_2 + Z_1} , \quad \frac{\rho Z_2 - \rho_1 Z_1}{\rho Z_2 + \rho_1 Z_1} .
\]

On the other hand, reflection and transmission of an ultrasonic guided wave are subject not to the specific acoustic
impedance but rather to the characteristic acoustic impedance \( Z_n = A_n \rho_w v_w \), where \( A_n \) is the cross-sectional area of the waveguide. The phenomena using the characteristic acoustic impedance can be described analogously based on the transmission line model of the electromagnetic waves. Since the wave speed of the T(0,1) mode guided wave do not vary with the wall thickness, the reflection and transmission coefficients, respectively, of the T(0,1) mode guided wave at a wall thinning are described only in the cross-sectional areas as follows:

\[
R_{12} = \frac{A_2 - A_1}{A_2 + A_1}, \quad t_{12} = \frac{2A_2}{A_2 + A_1}.
\]

The above equations indicate the reflection and transmission coefficients for an abrupt change in wall thickness. When the cross-sectional area changes gradually along the axial direction, as shown in Fig. 1, the total reflection coefficient \( R \) can be described by integrating the reflection and transmission coefficients of the subdivided elements throughout the entire region of the defect, as follows:

\[
R = \sum_{n=1}^{N} \prod_{m=1}^{n-1} (r_{m+1} \cdot R_{nm+1}) \times \exp\{i(\omega t - 2(n - 1)k \Delta z)\},
\]

where \( N, \omega, t, k, \) and \( \Delta z \) are number of subdivisions, frequency, time, wave number, and axial length of subdivisions, respectively.

3. Examples of Calculations

In this section, calculation results were presented in order to clarify the reflection and transmission phenomena qualitatively. The calculation condition was the same as that for the FEM calculation\(^{12}\) carried out by Carandente et al., and the calculation results were compared with the results of the FEM calculation for the validation of the calculation model described in eq. (3). A steel pipe having a thickness of 5.5 mm and an outer diameter of 76.2 mm was used. A tapered defect was introduced into the pipe specimen as shown in Fig. 2. The taper was 60 mm long and had 50% wall loss. Two transmission and reception points (TR 1 and TR 2 in Fig. 2) were located 300 mm from the taper in the right and left directions, respectively. \( N \) and \( \Delta z \) in eq. (3) were set to be 1000 and 60 \( \mu \)m, respectively. The frequency and velocity of the T(0,1) mode guided wave were 70 kHz and 3120 m/s, respectively. Two cycles of a Hanning-windowed sine wave were used for the generation of the guided wave.

Figures 3(a) and 3(b) show the calculation results for the time domain signals generated and detected at TR 1 and TR 2, respectively. First, the obtained waveforms coincided very well with the FEM calculations.\(^{12}\) The two isolated wave packets were clearly confirmed in both cases. Since the time difference between the two wave packets is the same as the round trip time of the guided wave in the taper, the reflections, at first glance, would be expected to occur only at the both ends of the taper. However, the calculation model shows that the reflections occurred at all locations in the tapered region, including both ends of the taper. It was also confirmed that the signal amplitudes at the thinner ends of the taper were approximately two times larger than those at the thicker ends of the taper in both cases as shown in Figs. 3(a) and 3(b). Here, these mechanisms of the reflection are explained using the illustration shown in Fig. 4. Reflections
occur at all locations in the tapered region, as shown in Fig. 4(a). Therefore, the reflected signal-elements must be generated continuously at all locations in the tapered region, as shown in Fig. 4(b). In Fig. 4(b), we show that the amplitude of each signal-element gradually increases with the decrease in thickness. The reason for these phenomena can be explained simply using eq. (3) as follows. The factor of the decrease in thickness. The reason for these phenomena can be explained simply using eq. (3) as follows. The factor of the direct product, \( \prod(f_{m+n+1} / f_{m+n}) \), in eq. (3) is only slightly less than 1 because the transmission coefficient between any pair of adjacent subdivided elements is only slightly less than 1 when the inclination angle of the taper is very small. This means that the direct product in eq. (3) is not a dominant factor in determining the total reflection coefficient of the entire taper. On the other hand, the reflection coefficients among the subdivided elements gradually increased with the decrease in the cross-sectional area of the subdivided elements. Furthermore, the factor \( f_{m+n+1} / f_{m+n} \) in eq. (3) at the thicker end is twice as small as that at the thinner end because the wall thickness in the thicker end is twice as thick as that at the thinner end (see eq. (2)). That is, the numerator of eq. (2) is always the same value but the denominator changes proportionally with the wall thickness. The explanation above is the main reason for the phenomena involving the amplitudes of the signal-elements, as shown in Fig. 4(b). Except for the signal-elements generated near the thicker and thinner ends, the signal-elements in the middle part vanish because of the interference of the positive and negative phases of the signals, as shown in Fig. 4(b). Therefore, the observed waveform can eventually be obtained as shown in Fig. 4(c). An asymmetric phase difference were observed between Figs. 3(a) and 3(b). These phenomena are due to the phase inversion at a reflection from a larger cross-sectional area to a smaller cross-sectional area (see eq. (2)).

4. Experiments for Validation of the Calculation Model

Figure 5(a) shows the experimental setup. Aluminum pipes having an outer diameter of 60.5 mm and a thickness of 3.9 mm were used. A piezoelectric ring-shaped sensor system having 16 transducer-elements was used to generate and detect the T(0,1) mode guided waves. The center frequency \( f \) was varied from 24 kHz to 50 kHz in 2 kHz steps. A tunable filter (NF 3628) was applied to the received signals in order to pass the frequency component \( f \pm 5 \text{kHz (48 dB/octave).} \) An axisymmetric tapered defect was introduced in each pipe, as shown in Fig. 5(a). The depth profile of the wall thinning of the axisymmetric tapered defect is shown in Fig. 5(b). The wall thinning in all of the pipes was to a depth of 2 mm, and the thickness of the remaining wall was 1.9 mm. The widths \( w \) of the tapered regions were set to be 0, 15, 30, 45, and 60 mm.

Figures 6 and 7 show the frequency variations of the time domain signals of both the experimental and calculated results for the taper widths \( w = 15 \) and 45 mm, respectively. In all the calculations, the number of subdivisions, \( N \), in eq. (3) was set to be 1000. It was confirmed that all of the experimentally obtained waveforms agreed excellently with the corresponding calculation results. The reflection coefficients as a function of frequency for all of the pipes were summarized in Fig. 8. The lines and circles in Fig. 8 indicate the calculation and experimental results, respectively. All of the calculation results are principally the same when the horizontal axis is described as a function of the product of frequency and taper width, \( f_w \), instead of frequency \( f \). Here, \( f_w \) is added as the upper axis to each graph of Fig. 8. The experimental results were in excellent agreement with the calculation results. As for the rectangular step-down defect (\( w = 0 \text{ mm} \)), the reflection coefficient was independent of the frequency change.

5. Inverse Problem

5.1 Simple method for solving inverse problems and experimental verification

We propose a simple method for estimating the wall thinning shape using the above calculation model. In this method, the shape of the wall thinning was assumed to be axisymmetric and to be circular (radius of curvature \( r \)) along the axial direction, as shown in Fig. 9. The procedure for estimating the shape consists of the following three steps:

1. Obtain experimentally the reflection coefficients \( \hat{R}(f) \) at the wall thinning as a function of frequency.
(2) Calculate the reflection coefficients $R(f, w, d)$ recursively for different widths $w$ and depths $d$ of wall thinning in their projected ranges.

(3) Determine the pair of $w$ and $d$ as values estimated while the difference between $E$ and $R(f, w, d)$ is the smallest.

Figure 10 shows the locations of both the sensor system and the axisymmetric defects on the specimen pipes. Four steel pipes (outer diameter: 60.5 mm, thickness: 5.5 mm) were used as specimens. Each pipe has a different artificial wall thinning inside the pipe. The nominal dimensions of the wall thinnings that are being machined are listed in Table 1 (see Fig. 9 for $w$, $d$, and $r$). One of the four pipes (specimen 4) was prepared as a pipe without wall thinning for comparison. The axial lengths of the pipes were 2000 mm (the defect pipes) and 4000 mm (the pipe without defect), respectively. The experimental instruments and their settings were the same as described in the previous section.

In the following estimation, the experimental reflection coefficients $E(f)$ for frequencies at 30, 35, 40, 45, and 50 kHz were first obtained. Second, the $R(f, w, d)$ was recursively calculated in the projected ranges ($w = 20$ to 200 mm in 1-mm steps, and $d = 0.3$ to 2.0 mm in 0.01-mm steps) at the same five frequencies to find the pair of $w$ and $d$ as estimated values. The pair of $w$ and $d$ was simply determined when the following evaluation function took the smallest value:

$$EF(w, d) = \sum f [E(f) - C(R(f, w, d))]^2.$$  \hspace{1cm} (4)

5.2 Results

The frequency variations (30 to 50 kHz) of the RF signals for specimens 1 through 4 are shown in Figs. 11(a) through 11(d), respectively. Two large amplitude signals, which were also saturated in the measurement range, were found at propagation times before approximately 0.1 ms and after approximately 0.4 ms, respectively. The first large signals indicate the guided waves propagating a short distance (propagation distance: 30 mm) from the transmitter to the next receiver. On the other hand, the last large signals indicate the signals reflected from the left edges of the specimen pipes. Since the length of the pipe without defect was 4000 mm (the length from the sensor to the left edge was 1770 mm), no large signals were found at around 0.4 ms in the pipe without defect (Fig. 11(d)). Comparing to the other four frequencies, the relatively large spurious baseline signal was found at 30 kHz (Fig. 11(d)). This is due to the specific spurious signal due to the sensor used. The signals that arrived at around 0.3 ms are the signals from the wall thinnings. The signal amplitudes were confirmed to vary with the frequency. In Fig. 11(b), the signal amplitude at 35 kHz was very near to the noise level (see Fig. 11(d)) and it was too difficult to identify the isolated defect signal. This result indicates one of the unsafe situations in the practical use of the guided waves, i.e., the guided wave inspection might overlook a defect in a pipe when only a single frequency is used. The use of multiple frequencies might be important and essential to avoid these types of oversights. Figures 12(a) through 12(c) show the reflection coefficients as a function of frequency for three different wall thinnings, specimens 1 through 3, respectively. The circles and crosses in Fig. 12 show, respectively, the signal amplitudes and noise levels. The lines in Fig. 12 show the calculated reflection coefficients when the evaluation function, eq. (4), takes the smallest value. The pairs $(w's [\text{mm}]$ and $d's [\text{mm}]$) estimated for specimens 1, 2, and 3 were (41, 0.84), (67, 0.70), and (26, 0.25), respectively. The depth profiles estimated by the present method are shown in Figs. 13(a), 13(b), and 13(c) together with those measured by UT. The depth profiles
estimated for specimens 1 and 2 were confirmed to agree very well with those measured by UT. Conversely, for specimen 3, the estimated profile did not coincide with the measured profile, as shown in Fig. 13(c).

### 5.3 Discussion

The accuracy in the estimation of the depth profile is presumably dependent on the relation between the frequency range used and the axial length of the wall thinning. In the following, the accuracy of the estimation of defect sizing in

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**Fig. 8** Reflection coefficients as a function of frequency for different taper length, (a) \(w = 0\) mm, (b) \(w = 15\) mm, (c) \(w = 30\) mm, (d) \(w = 45\) mm, and (e) \(w = 60\) mm. The lines and circles indicate experimental and calculation results, respectively. The upper axes in (b) through (e) indicate the product of the frequency and the taper length, \(fw\). The reflection coefficient (\(w = 0\) mm) was confirmed to be independent to the frequency. All of the calculation results are the same with respect to \(fw\).

**Fig. 9** Axial cross section of the axisymmetric wall thinning being used in the experiments for inverse estimations of the depth profiles.

**Fig. 10** Axial locations of the ring-shaped sensor system and the wall thinning. The wall thinnings were fabricated inside the steel pipes.

**Table 1** Dimensions of wall thinning [nominal].

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>(w) (mm)</th>
<th>(d) (mm)</th>
<th>(r) (mm)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>1</td>
<td>113</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>1</td>
<td>451</td>
</tr>
<tr>
<td>3</td>
<td>120</td>
<td>1</td>
<td>1800</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>(\infty)</td>
</tr>
</tbody>
</table>
Fig. 11 Frequency variations of the RF signals for (a) specimen 1, (b) specimen 2, (c) specimen 3, and (d) specimen 4.

Fig. 12 Reflection coefficients as a function of frequency for (a) specimen 1 \((w = 30\text{\,mm [nominal]})\), (b) specimen 2 \((w = 60\text{\,mm [nominal]})\), and (c) specimen 3 \((w = 120\text{\,mm [nominal]})\). The lines, circles, and crosses indicate the calculated and experimental results and the noise levels, respectively.

Fig. 13 Depth profiles for (a) specimen 1, (b) specimen 2, and (c) specimen 3. Each thick line indicates the estimation profile of each specimen. The circles indicate profiles measured by UT.
terms of the evaluation function was discussed. Moreover, this section has revealed a reason for the disagreement between the experiment and the calculation, as shown in Fig. 13(c).

Figures 14(a), 14(b), and 14(c) show gray-scale representations of the differences between the experimental and calculated reflection coefficients evaluated by eq. (4) for specimens 1, 2, and 3 as functions of both \( w \) and \( d \), respectively. The gray scale was shown in terms of the \( z \)-th power of ten. In Fig. 14(a), only the one local minimum was found at \( w = 41 \) mm and \( d = 0.84 \) mm and the figures were none other than the sizes of the estimation. The value of the evaluation function at the minimum was \( 8.33 \times 10^{-6} \). The above determination process for sizing the wall thinning is reasonably well understood. In Fig. 14(b), we found four local minimums. Indexes \( \text{①, ②, ③, and ④} \) in Fig. 14(b) indicate the four local minimums at \( (w \ [\text{mm}], \ d \ [\text{mm}]) = (31, 0.20), (67, 0.70), (115, 1.16), \) and \( (161, 1.74), \) respectively. The values of the evaluation function at these four local minimums were \( 3.51 \times 10^{-4}, 9.90 \times 10^{-6}, 1.73 \times 10^{-4}, \) and \( 2.45 \times 10^{-4} \), respectively. The local minimum \( \text{②} \) was confirmed to be at least 20 times smaller than the other local minimums, and thus the pair of \( w \) and \( d \) was determined to be \( 67 \) mm and \( 0.70 \) mm, as shown in section 5.2. Finally, three local minimums appeared in Fig. 14(c). Indexes \( \text{①, ②, and ③} \) in Fig. 14(c) indicate the three local minimums at \( (w \ [\text{mm}], \ d \ [\text{mm}]) = (25, 0.26), (71, 1.02), \) and \( (127, 2.02), \) respectively. The values of the evaluation function at these minimums were \( 5.01 \times 10^{-5}, 1.21 \times 10^{-4}, \) and \( 2.72 \times 10^{-4} \), respectively. In this case, the local minimum \( \text{①} \) is the smallest. However, the estimated pair of \( w \) and \( d \) disagreed with the size measured by UT, as described in Fig. 13(c).

Here, in specimen 3, the second smallest local minimum \( \text{②} \) was approximately twice as large as the minimum \( \text{①} \). We must take into consideration this difference, which was relatively small compared to the difference of twenty times observed for the case of specimen 2. The depth profiles indicated by the three local minimums and that measured by UT are shown in Fig. 15. The line with circles shows the depth profile measured by UT. The lines \( \text{①, ②, and ③} \) in Fig. 15 are the depth profiles corresponding to the local minimums in Fig. 14(c). Comparing the three lines with the UT measurement, the line \( \text{②} \) coincides well with the UT measurement. Conversely, the other two lines are much different from the UT measurement. The estimation appears to have succeeded, if the second local minimum \( \text{②} \) takes the first minimum in the evaluation function. Next, we consider the reason for this disagreement. Figure 16 shows both the reflection coefficients calculated with the three local minimums and the experimental values. The circles in Fig. 16 show the experimental results. In the frequency range of from 30 to 50 kHz, we can confirm that all three lines \( \text{①, ②, and ③} \),
Here, we can reach an important outcome to improve the amplitudes in the frequency range following the main lobe. The horizontal axis is described in depth of 1 mm. The five circles indicate the experimental results. The thin, thick, and dotted lines, respectively, indicate the reflection coefficients calculated corresponding to the local minimums, ①, ②, and ③. All of the reflection coefficients were confirmed to take values close to the set of the experimental reflection coefficients.

③ take similar values to the experimental results depicted by the circles. This is why three local minimums exist in Fig. 14(c). Conversely, the three lines can be easily distinguished by comparing the lines in the frequency range of less than 30 kHz because these lines take very different values in the frequency range of less than 30 kHz. This means that the estimation accuracy must be improved when a wider frequency range (especially in the lower frequency range) will be used. Moreover, there is an important relation between the axial length of the defect and the frequency of the guided wave to improve the estimation accuracy. Figure 17 shows the reflection coefficient of the circular, concave, axisymmetric defect with a depth of 1 mm in a steel pipe having an outer diameter of 60.5 mm and a thickness of 5.5 mm. The horizontal axis was described as a function of \( f_w \) (kHz-mm). All of the circular concave defects having a depth of 1 mm take the same reflection coefficient when the horizontal axis is described in \( f_w \), as described before. The reflection coefficient consists of one main lobe and two side lobes, as shown in Fig. 17. The main lobe has the largest amplitude in the lowest frequency range from DC to around 2000 kHz-mm, and the side lobes have relatively low amplitudes in the frequency range following the main lobe. Here, we can reach an important outcome to improve the accuracy of the estimation. That is to say, the reflection coefficients measured around the peak of the main lobe (1080 kHz-mm) is very sensitive for determining the shape of the defect because the evaluation function, eq. (4), must take the smallest value. For example, when the width is 20, 40, and 60 mm, the most effective range of frequency is around 54, 27, and 18 kHz, respectively. Therefore, when a defect having an unknown axial length ranging from 20 to 60 mm might exist, a frequency range of approximately 18 to 54 kHz is needed in order to obtain more accurate estimation results. Therefore, we had to use the frequency range from at least 15 kHz to 50 kHz to accurately determine the defect size of specimen 3 in the estimation process because the axial defect length of specimen 3 exceeded 70 mm. However, in the present procedure, a frequency range of from 30 to 50 kHz had been used to determine the dimension of the wall thinning, as shown in the previous section, which is presumably the primary reason why the estimation failed. Fundamentally, the peak frequency of the main lobe is determined by the interference between the two equivalent wave packets, i.e., one is the equivalent wave packet reflected at the gradual step-down region of the wall thinning and the other is the equivalent wave packet reflected at the gradual step-up region of the wall thinning. Both the equivalent distance between the step-down and the step-up positions, which is slightly shorter than the full width of the wall thinning \( w \), and the phase inversion at the step-down region determines the peak frequency of the main lobe.

6. Conclusion

Reflection coefficients of the T(0,1) mode guided wave at axisymmetric tapered defects have been investigated. Different frequency dependences were observed depending on the axial profile of the defect. It was confirmed that the frequency dependences of the reflection coefficients at the axisymmetric tapered defects were in excellent agreement with those obtained by the mathematical reflection model using the characteristic acoustic impedance. The mathematical model was validated experimentally and was very useful for calculating of the reflection coefficient from both a shape of an axisymmetric defect and the frequency. It was also shown that the mathematical model explained well the waveform forming mechanism of the RF time domain signals. This paper also proposed a novel approach for estimating the approximate dimensions (depths and axial lengths) of defects using the mathematical model. Discussion of the estimation results was carefully carried out, which showed that the appropriate frequency range corresponding to the axial defect length was essential for the successful estimation.

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