Relation of \( n \)-Value to Critical Current for Local Sections and Overall Sample in a SmBCO Coated Conductor Pulled in Tension

Shojiro Ochiai\(^1\), Hiroshi Okuda\(^2\), Shinji Nagano\(^2\),\(^*,\) Michinaka Sugano\(^3\), Sang-Song Oh\(^4\), Hong-Soo Ha\(^4\) and Kozo Osamura\(^5\)

\(^1\)Elements Strategy Initiative for Structural Materials, Kyoto University, Kyoto 606-8501, Japan
\(^2\)Department of Materials Science and Engineering, Kyoto University, Kyoto 606-8501, Japan
\(^3\)High Energy Accelerator Research Organization (KEK), Cryogenics Science Center, J-PARC Center, Naka-gun, Ibaraki 319-1106, Japan
\(^4\)Korea Electrotechnology Research Institute, 28-1 Sangju-Dong, Changwon 641-120, Korea
\(^5\)Research Institute for Applied Sciences, Kyoto 606-8202, Japan

Under application of tensile stress to a SmBCO (SmBa\(_2\)Cu\(_3\)O\(_{7-x}\)) coated conductor sample consisting of series electric circuit of local sections, the relation of voltage–current curve, critical current and \( n \)-value of the sections to those of overall sample was studied. The change in critical current and \( n \)-value with increasing applied stress was different from section to section due to the difference in damage behavior of the SmBCO layer among the sections. When the difference in extent of damage among the sections was small, the voltages developed in all sections contributed to the voltage of overall sample. In this case, the critical current and \( n \)-value of overall sample were within the range of the highest and lowest values among the sections. On the other hand, when the damage in one section was far severer than that of other sections, the voltage developed in the most severely damaged section largely contributed to the overall voltage, and hence the voltage-current curves of the most severely damaged section were almost the same as those of overall sample. In this case, critical current of the overall sample was slightly higher and \( n \)-value of the overall sample was lower than the critical current and \( n \)-value of the most severely damaged section. Accordingly, the decrease in \( n \)-value with decreasing critical current in overall sample was sharper than that in sections. This phenomenon was accounted for by the increase in shunting current at cracked part at higher voltage in the most severely damaged section. [doi:10.2320/matertrans.MBW201301]

(Received September 25, 2013; Accepted November 5, 2013; Published December 13, 2013)

**Keywords:** SmBCO coated conductor, crack, critical current, \( n \)-value, sample length

1. Introduction

Superconducting composite tapes are subjected to thermal, mechanical and electromagnetic stresses during fabrication and in service. When the subjected stress/strain is high, the superconducting layer/filament is mechanically damaged, mainly by cracking.\(^1\)\(^\sim\)\(^16\) As a result, superconducting properties such as critical current \( I_c \) and \( n \)-value are seriously reduced in both coated\(^1\)\(^\sim\)\(^9\) and filamentary\(^10\)\(^\sim\)\(^16\) tapes. Usually, damage takes place non-uniformly within a sample.\(^1\)\(^\sim\)\(^4\)\(^4\) As the electric circuit of a sample is composed of series circuits of sections, the superconducting property of the sample varies, depending on the extent of damage in the sections. For safe and reliable design, it is required to describe the relation of extent of damage to superconducting property in sections and the relation of superconducting property of sections to that of overall sample.

In the present work, we measured \( V \)(voltage)–\( I \)(current) curve, \( I_c \) and \( n \)-value of sections and overall sample for a SmBa\(_2\)Cu\(_3\)O\(_{7-x}\) (SmBCO) coated conductor pulled in tension at 77 K under various applied stresses. The experimental result showed the following features. (1) The critical current and \( n \)-value were reduced by the stress-induced damage in SmBCO layer in both sections and overall sample. (2) When the difference in the damage extent among the sections was small, \( I_c \) and \( n \)-value of overall sample were within the range of the highest and lowest values among the sections. (3) When the damage in one section was far severer than that of other sections, \( I_c \) of the overall sample was slightly higher than that of the most severely damaged section, while \( n \)-value of the overall sample was lower than that of the most severely damaged section. In this case, the decrease in \( n \)-value with decreasing \( I_c \) in overall sample was sharper than that in sections. The features mentioned above were accounted for by modeling analysis. These results are reported in this paper.

2. Experimental Procedure

A SmBCO coated conductor tape, consisting of Cu(thickness: \(~20 \mu m\))/Ag(\(~3 \mu m\))/SmBCO(\(~2 \mu m\))/LaMn-O\(_3\)(\(~30 nm\))/MgO(\(~70 nm\))/Y\(_2\)O\(_3\)(\(~7 nm\))/Al\(_2\)O\(_3\)(\(~50 nm\))/Hastelloy(\(~75 \mu m\))/Cu(\(~20 \mu m\)), fabricated at Korea Electrotechnology Research Institute (KERI),\(^17\) was used for study. The width of the tape was 4 mm. The \( V \)–\( I \) curves were measured for the sections (S1, S2 and S3) and overall sample in Fig. 1, where each section has a length \( L = 1.5 \) cm and overall sample has a length \( L = 4.5 \) cm. Due to the difference

\[ \text{Fig. 1 Schematic representation of the test sample with a length } L = 4.5 \text{ cm, composed of three local sections (S1, S2 and S3) with a length } L = 1.5 \text{ cm. Critical current } I_c \text{ and } n \text{-value were measured for each section and overall sample.} \]

\( ^* \)Graduate Student, Kyoto University
in damage behavior among the sections, the strains of sections are different to each other but stress is common for all sections and overall sample. Due to this reason, stress is used instead of strain as a measure of applied mechanical condition.

By using the test piece shown in Fig. 1, we measured \( V-I \) curves of sections (S1, S2, S3) and overall sample under applied tensile stress at 77 K in a self magnetic field. The critical current \( I_c \) was estimated with an electric field criterion of \( E_c = 1 \mu V/cm \), corresponding to a critical voltage \( V_c = LE_c = 1.5 \mu V \) for the sections (S1, S2, S3) with a length of \( L = 1.5 \text{ cm} \), and corresponding to \( V_c = 4.5 \mu V \) for the overall sample with a length of \( L = 4.5 \text{ cm} \). The \( n \)-value was estimated by fitting the measured \( V-I \) curve to the form of \( V \propto I^n \) for the electric range of \( E = 0.1-10 \mu V/cm \), corresponding to \( V = 0.15-15 \mu V \) for the sections and \( V = 0.45-45 \mu V \) for overall sample.

### 3. Results and Discussion

#### 3.1 Measured critical currents (\( I_c \)) and \( n \)-values

Figure 2 shows the measured \( V-I \) curves of sections (S1, S2, S3) and overall sample at representative applied stresses \( \sigma_T = 497-552 \text{ MPa} \). The critical current and \( n \)-value were obtained from the \( V-I \) curves with the procedure stated in Section 2. The results are shown in Fig. 3. Figure 3(a) shows the critical current \( I_c \) and (b) shows \( n \)-value for the sections (S1, S2 and S3) and overall sample, plotted against tensile stress \( \sigma_T \). (c) Shows the plot of \( n \)-value against critical current \( I_c \).

1. Cracking of SmBCO layer did not take place at \( \sigma_T = 497 \text{ MPa} \). At other stresses \( \sigma_T = 528, 530, 535, 547 \text{ and} \)
552 MPa), it took place. When cracking took place, \( V-I \) curves shifted to lower current range (Fig. 2) and both of critical current \( I_c \) and \( n \)-value decreased with increasing applied stress \( \sigma_T \) (Fig. 3).

(2) At \( \sigma_T = 497 \) MPa where cracking of the SmBCO layer did not take place and at \( \sigma_T = 528 \) MPa where cracking took place but not extended largely, the voltages developed in all sections contributed to the voltage of overall sample (Figs. 2(a) and 2(b)). At these stresses, the \( V-I \) curves, \( I_c \) and \( n \)-value of the sections are not so much different to each other in comparison with those at higher stresses shown in Figs. 2(c)–2(f). The \( I_c \) and \( n \)-value of overall sample were within the range of the highest and lowest values among the sections (Figs. 3(a) and 3(b)).

(3) At higher stresses (\( \sigma_T \geq 530 \) MPa), one section (S1) was seriously degraded in comparison with other sections. The \( I_c \) and \( n \)-value of the most severely damaged section (S1) were lower than those of other sections (S2, S3). The \( V-I \) curve of S1 was almost overlapped with that of overall sample up to around \( V = 20 \) µV (Figs. 2(c)–2(f)), though the voltages developed at S2 and S3 contributed slightly to the voltage of overall sample. The overlapping of \( V-I \) curve of S1 with that of overall sample means that the \( I_c \) of overall sample (\( V_c = 4.5 \) µV) is slightly higher than that of S1 (\( V_c = 1.5 \) µV). Actually, the measured \( I_c \) values of overall sample at \( \sigma_T = 530, 535, 547 \) and 552 MPa were 122, 77, 49 and 18 A, which were slightly higher than those of S1; 118, 73, 46 and 16 A, respectively, while the difference in \( I_c \) between S1 and overall sample was not clearly visible in the scale of Fig. 3(a). In contrast, the \( n \)-values of overall sample at these stresses were visibly lower than those of the most severely damaged S1, as shown clearly in Fig. 3(b).

(4) As shown in (3) above, when damage took place severely in one section, \( I_c \) of the overall sample was slightly higher than that of the most severely damaged section but \( n \)-value of the overall sample was lower than that of the most severely damaged section whose \( n \)-value was lowest among the sections. As a result, the decrease in \( n \)-value with decreasing \( I_c \) in overall sample was sharper than that in sections (Fig. 3(c)).

3.2 Model for analysis

Figure 4 shows a schematic representation of (a) current path and (b) electrical circuit under an existent crack, in which \( I_s \) and \( I_{Sm} \) refer to the shunting current at the SmBCO-cracked part and SmBCO-ligament part-transported current, respectively.

In the transverse cross-section of which a partial crack exists in SmBCO layer (Fig. 4), the cracked part that has lost superconductivity and ligament (non-cracked) part that keeps superconductivity constitutes of a parallel circuit. We define the ratio of cross-sectional area of cracked part to the total cross-sectional area of the SmBCO layer as \( f \). The ligament part with an area ratio \( 1 - f \) transports current \( I_{Sm} \). The cracked part with an area ratio \( f \), current \( I_c = (1 - f)I_{Sm} \) shunts into Ag and Cu. In the shunting circuit, the resistances at the SmBCO-Ag interface, Ag–Cu interface and resistances in Ag and Cu are included in the total resistance \( R_t \) (Fig. 4(b)). The voltage, developed in the ligament part that transports current \( I_{Sm} \), is noted as \( V_{Sm} \) in Fig. 4(b). The voltage \( V_s = I_sR_s \), developed in the cracked part by shunting current \( I_c \), is equal to \( V_{Sm} \) since the ligament- and cracked parts constitute of a parallel circuit. Here, we denote the crack width as \( s \), critical current in non-cracked state as \( I_{cob} \) and \( n \)-value in non-cracked state as \( n_0 \). The \( V-I \) relation of cracked section \( (L = 1.5 \) cm) is expressed as\(^{5,6,14}\)

\[
V = \frac{E_c}{C_0} \left( \frac{I}{I_{cob}} \right)^{n_0} + V_s \tag{1}
\]

\[
I = I_{Sm} + I_s = I_{cob}(1 - f) \left( \frac{L}{s} \right)^{1/n_0} \left[ \frac{V_s}{E_c} \right]^{1/n_0} + \frac{V_s}{R_t} \tag{2}
\]

Using eqs. (1) and (2), we can obtain the values of \( (1 - f)(L/s)^{1/n_0} \) and \( R_t \) by curve fitting to the measured \( V-I \) curve, and, once they are estimated, the SmBCO ligament part-transported current \( I_{Sm} \) and shunting current \( I_c \) can be obtained, as has been shown in our preceding work.\(^{5,6,14}\) In calculation with eqs. (1) and (2), the values of \( I_c \) and \( n \) at \( \sigma_T = 497 \) MPa just below the onset stress of cracking were used as \( I_{cob} = 180, 171 \) and 183 A, and \( n_0 = 35.2, 36.1 \) and 32.2 for S1, S2 and S3, respectively.

3.3 Synthesis of \( V-I \) curves of overall sample by using the estimated values of \( (1 - f)(L/s)^{1/n_0} \) and \( R_t \) of sections

The values of \( (1 - f)(L/s)^{1/n_0} \) and \( R_t \) of cracked S1, S2 and S3 at \( \sigma_T = 528, 530, 535, 547 \) and 552 MPa, obtained by
fitting the experimentally measured $V$-$I$ curves to eqs. (1) and (2) are shown in Fig. 5. To examine the accuracy of the obtained values of $(1 - f)(L/s)^{1/n_0}$ and $R_i$, the $V$-$I$ curves of S1, S2 and S3 at each $\sigma_T$ were back-calculated by eqs. (1) and (2) with the obtained values, and $I_c$ and $n$-value were obtained from the calculated $V$-$I$ curves. The obtained values of $I_c$ and $n$, together with experimental results, are shown in Figs. 6(a) and 6(b). The experimental results are reproduced satisfactorily.

The sections (S1, S2 and S3) constitute of a series electric circuit in the overall sample. Noting the currents of S1, S2 and S3 as $I_1$, $I_2$ and $I_3$, respectively, and the current of overall sample as $I$, $I = I_1 = I_2 = I_3$ is held since imposed current is common for all sections and overall sample. Noting the voltages developed in S1, S2 and S3 as $V_1$, $V_2$ and $V_3$, respectively, and the voltage developed in the overall sample as $V$, $V = V_1 + V_2 + V_3$ is held since the voltage developed in the overall sample is the sum of voltages of S1, S2 and S3. Accordingly, using eqs. (1) and (2), the current $I$ and voltage $V$ of overall sample are expressed as

$$I = I_1 = I_{o_dj} \left[ (1 - f) \left( \frac{L}{s} \right)^{1/n_0} \right] + \frac{V_{s,j}}{E_c L} \left( \frac{I}{I_{o_dj}} \right)^{1/n_0} + \frac{V_{s,j}}{R_{s}} \quad (i = 1, 2, 3) \tag{3}$$

$$V = \sum_{i=1}^{3} E_c L \left( \frac{I}{I_{o_dj}} \right)^{n_0} + V_{s,j} \quad \tag{4}$$

where the subscript $i$ (=1, 2, 3) of $I_{o_dj}$, $(1 - f)(L/s)^{1/n_0}$, $n_0$, $V_s$ and $R_s$ refer to the values of S1, S2 and S3, respectively. Equations (3) and (4) can be used for synthesis of $V$-$I$ curve of cracked overall sample ($\sigma_T = 528, 530, 535, 547$ and 552MPa). The $V$-$I$ curve of non-cracked overall sample ($\sigma_T = 497$MPa) can be synthesized by

$$V = \sum_{i=1}^{3} E_c L \left( \frac{I}{I_{o_dj}} \right)^{n_0} \quad \tag{5}$$

The values of $I_{o_dj}$, $(1 - f)(L/s)^{1/n_0}$, $n_0$, $V_s$ and $R_s$ are already known, as have been shown. Substituting these known values and a given value for $I$, $V_{s,d}$ in cracked state can be calculated by eq. (3). Then substituting $V_{s,j}$ and $I$ into eq. (4), we can have $V$-$I$ curve for cracked overall sample. The $V$-$I$ curve of non-cracked overall sample can be calculated by substituting the known values of $I_{o_dj}$ and $n_0$, into eq. (5). The measured and calculated $V$-$I$ curves of the overall sample at each applied stress are compared in Fig. 7. From the synthesized $V$-$I$ curve of the overall sample, the $I_c$ and $n$-value were obtained and were compared with the experimental results, as shown in Fig. 8. The measured $V$-$I$ curves (Fig. 7), $I_c$ and $n$-value (Fig. 8) of the overall sample are reproduced satisfactorily by the present approach. These results indicate that the values of $(1 - f)(L/s)^{1/n_0}$ and $R_i$ are proper and are useful to derive shunting current.

### 3.4 Analysis of $n$-$I_c$ relation of overall sample

As has been shown in Fig. 2, the $V$-$I$ curves of the overall sample ($\sigma_T \geq 530$MPa) were almost overlapped with the $V$-$I$ curves of the most severely damaged section (S1). Also, as has been shown in Fig. 3(c), the decrease in $n$-value with decreasing critical current in the overall sample
Fig. 7 Measured and calculated $V$-$I$ curves of the overall sample at $\sigma_T = 547$ MPa. The solid curves show the experimental results. The broken curves show the calculated curves by substituting the values of $I_{c0}$, $n_0$, $(1-f)(L/s)^{1/n_0}$ and $R_s$ of S1, S2 and S3 into eqs. (3) and (4).

Fig. 8 Comparison of the measured (a) critical current $I_c$ and (b) $n$-value of the overall sample with the obtained values from the calculated $V$-$I$ curves shown in Fig. 7.

$(\sigma_T \geq 530$ MPa) was severer than that in sections. In this subsection, the reason for this is discussed.

The $V$-$I$ curve can be calculated by substituting the known values of $(1-f)(L/s)^{1/n_0}$ and $R_s$ into eqs. (1) and (2). At the same time, the SmBCO-ligament-transported current $I_{sm}$ is calculated by $I_{c0}(1-f)(L/s)^{1/n_0}[V_c/(E_cL)]^{1/n_0}$ and the shunting current $I_s$ is calculated by $V_c/R_s$. Figure 9(a) shows an example of calculated change in $I_{sm}$ and $I_s$ as a function of $I_c$ in which the values of $(1-f)(L/s)^{1/n_0}$ and $R_s$ of the most severely damaged section S1 at $\sigma_T = 547$ MPa were input in calculation. Shunting current $I_s$ is low at low $I_c$ and (low $V$), while the $I_{sm}$ is on high level. The total transport current $I$ ($= I_{sm} + I_s$) is almost determined by $I_{sm}$ in low $I_c$ (low $V$) range. On the other hand, $I_s$ increases largely beyond around $I_c$ at $V_c = 1.5 \mu$V in contrast to the gradual increase in $I_{sm}$.

Figure 9(b) shows the $V$-$I_{sm}$ and $V$-$I$ curves in logarithmic scale. Shunting current $I_s$ given by $I-I_{sm}$ increases with increasing voltage $V$. When the $V$-$I$ curve of S1 with $L = 1.5$ cm is overlapped with that of the overall sample with $L = 4.5$ cm, the critical current of the overall sample at $V = V_c = 4.5 \mu$V is higher than that of S1 at $V = V_c = 1.5 \mu$V due to the increased $I_c$, as indicated in Fig. 9(b). This accounts for the slightly higher $I_c$ value of overall sample in comparison with $I_c$ value of S1 (Subsection 3.1). The $n$-value corresponds to the average slope of $V$-$I$ curve in logarithmic scale for a defined range of $V$ ($0.15-15 \mu$V for S1 with $L = 1.5$ cm and $0.45-45 \mu$V for overall sample with $L = 4.5$ cm). As $I_c$ increases with $V$, the $n$-value of overall sample is low in comparison with the $n$-value of S1. In this way, $n$-value is reduced with increasing $L$ under the condition of overlapping of the $V$-$I$ curve, due to the enhanced shunting current in cracked sample. As has been shown in Fig. 2, the $V$-$I$ curves of overall sample $\sigma_T = 530, 535, 547$ and 552 MPa are almost overlapped with those of the most severely damaged section (S1). The experimental result in Fig. 3(b), showing that the $n$-values of overall sample at $\sigma_T = 530, 535, 547$ and 552 MPa are lower than those of the most severely damaged section (S1), are accounted for from a viewpoint of the enhancement of shunting current with increasing $V$ in the most severely damaged section.

Figure 10 shows the measured $n-I_c$ relation of overall sample and that of sections S1, S2 and S3 for reference, in
comparison with the calculated ones with two models (noted as Model(S1+S2+S3) and Model(S1)). Model(S1+S2+S3) refers to the model used in synthesis of $V-I$ curve (Fig. 7) of overall sample with the data of all sections of S1, S2 and S3. The $I_c$ and $n$-value of the overall sample calculated by Model(S1+S2+S3) shown in Fig. 8 are plotted in Fig. 10. Model(S1) refers to a model which assumes that the $V-I$ curve of overall sample is completely controlled by that of the most severely damaged section (S1) and the other sections (S2, S3) do not contribute to the synthesis of the $V-I$ curve of the overall sample. In Model(S1), the $I_c$ and $n$-value of overall sample were calculated by using the $V-I$ curve of S1 at $V_c = 4.5 \mu$V and $V = 0.45 - 45 \mu$V, respectively, as shown in Fig. 9(b).

In Fig. 10, the feature of the experimental results at $\sigma_T = 528$ MPa (when the extent of damage is not so much different among the sections, the $I_c$ and $n$-value of overall sample lies within the range of the highest and lowest values among the sections (Figs. 3(a) and 3(b)) was well reproduced by Model(S1+S2+S3). Model(S1) gives too much under-estimation for the $n$-value of overall sample. The feature of the experimental results at $\sigma_T \geq 530$ MPa (when one section is far more damaged than the other sections, the $I_c$ and $n$-value of overall sample lies beneath the range of the highest and lowest values among the sections and overall sample were within the range of the highest and lowest values among the sections).

As shown in the present work, (a) when the damage extent among the sections is small, the $I_c$ and $n$-value of overall sample are within the range of the highest and lowest values among the sections and (b) when one section is far more damaged than the other sections, $n$-value of overall sample is lower than the lowest $n$-value among all sections and the decrease in $n$-value with decreasing $I_c$ in the overall sample is sharper than that in any section. These phenomena are accounted for from the increase in shunting current at higher voltage (Fig. 9), reducing $n$-value for longer sample.

4. Conclusions

(1) The stress-induced damage caused a shift of $V-I$ curve to lower current region, and caused reduction in $I_c$ and $n$-value in all sections and overall sample.

(2) The change in $V-I$ curve, $I_c$ and $n$-value with increasing applied stress was different from section to section due to the difference in damage behavior of the SmBCO layer among the sections. As a result, there arose two cases shown below in (3) and (4).

(3) When the damage extent among the sections was small, the voltages developed in all sections contributed to the voltage of overall sample. In this case, $I_c$ and $n$-value of overall sample were within the range of the highest and lowest values among the sections.

(4) When the damage in one section was far severer than that of other sections, the voltage developed in the most severely damaged section largely contributed to the overall voltage and the $V-I$ curve of the most severely damaged section was almost the same as that of the overall sample. In this case, $I_c$ of the overall sample was slightly higher than that of the most severely damaged section and $n$-value of the overall sample was lower than that of the most severely damaged section. In this case, the $n-I_c$ relation of the overall sample was lower than that of any sections.

(5) The results in (3) and (4) were described by combining a current shunting model at the cracked part with a series circuit model.

Acknowledgements

The authors wish to express their gratitude to The Ministry of Education, Culture, Sports, Science and Technology, Japan for the grant-in-aid for scientific research (No. 22360281).

REFERENCES

6) S. Ochiai, H. Okuda, T. Arai, S. Nagano, M. Sugano and W. Prusseit:
Relation of $n$-Value to Critical Current for Local Sections and Overall Sample in a SmBCO Coated Conductor Pulled in Tension

Cryogenics 51 (2011) 584–590.