Influence of Water Pulsation with Different Frequency and Amplitude on Orbit of a Particle Placed on a Fixed Screen

Tatsuya Oki¹, Taeko Hazumi¹, Yoko Umemiya¹ and Mikio Kobayashi²

¹National Institute of Advanced Industrial Science and Technology, Tsukuba 305-8569, Japan
²Japan Oil, Gas and Metals National Corporation, Kawasaki 212-8554, Japan

Gravity separation techniques such as jigging are based on the difference in the settling velocities of particles in water, which depends on the specific gravity, size, and shape of the particles. When a particle falls in stationary water, it is known that its settling velocity in the acceleration period before it reaches its terminal velocity depends on its specific gravity. Therefore, if one jigging cycle were completed before the particle reaches its terminal velocity, separation based on the difference in particle specific gravity would be promoted as the jigging frequency increases. However, there are few reports discussing whether the acceleration period in stationary water could be applied to an unsteady jigging movement. In this study, we used particles of around 1 mm, which are small particles for a jig, and investigated the relation between jigging frequency and particle movement. We have concluded from the results that although the reason is incorrect, the above classical hypothesis reflects the actual phenomenon, since the effect calculated in this study almost agrees with estimations obtained using the hypothesis developed by Gaudin.

(Received December 16, 2008; Accepted October 2, 2009; Published December 2, 2009)

Keywords: jig, settling velocity, equal settling ratio, particle size, gravity separation

1. Introduction

The technologies for separating particles in water based on specific gravity or particle size are likely to be increasingly applied to recycling or other industrial fields, since particle separation can be carried out without using any heavy medium or chemicals and with a low environmental load. These technologies carry particles in different directions after generating particle beds in water, and the most fundamental movement exploited by these methods is the different particle velocities in water. Since particle velocity in water depends on the specific gravity, size, and shape of the particles, separation by using specific gravity cannot be realized if the sizes are not adjusted in advance so that they fall within a suitable range, even if the difference in shape can be ignored. In a device that separates particles solely in terms of differences in settling velocity, such as an elutriator, the particle size range that needs to be adjusted in advance is called the “equal settling ratio”. Although the movement in water of particles with a steady motion can be estimated relatively easily, it is difficult to estimate accurately the movement of particles whose motion is unsteady. For example, the equal settling ratios of particles with different specific gravities falling in vertically oscillating water are not known. In previous reports¹-³ we have confirmed by experiments and calculations that the equal setting ratios in vertically oscillating water are smaller than that in stationary water.

In contrast, the jigging movement, which catches particles on a screen in each oscillation cycle, does not obey the concept of the equal settling ratio, because particle separation occurs as the result of a combination of several factors including hindered settling and consolidation tricking. Since the movement of particles in jigging water is complicated, strict theoretical prediction is difficult. Although there are some reports that used calculations to outline particle motion in jigging water, the separation properties are generally investigated experimentally in almost every case.⁴-⁶ However, some ideas that focus on fragmentary jig properties have already been proposed with a view to improving the separation accuracy.⁷ One well-known classical concept is that gravity separation accuracy improves when the jig frequency is high. This is because the velocity of particles falling in water during the acceleration period before the terminal settling velocity is reached strongly depends on the specific gravity of the particles. Figure 1 shows the change in particle velocity in the early falling period for three kinds of particles with different specific gravities and sizes.⁷ Even if the particles have the same terminal settling velocities (vm₁), the courses they take to reach the terminal settling velocity are different for small high specific gravity particles and large low specific gravity particles. Although “A”, which is a small high specific gravity particle, reaches its terminal settling velocity quickly, “B”, which is a large low specific gravity particle accelerates slowly. “C”, which has the same specific gravity as “A” but a smaller size traces a course to reach a low terminal settling velocity (vm₂). Therefore, both of the high specific gravity particles (A and C) can be separated from a low specific gravity particle (B) in time t₁. In other words, particles can be separated by the difference in their movements during the acceleration period if one jigging
cycle finishes before the particle reaches its terminal settling velocity, and then the accuracy of gravity separation would be improved compared with separation by the difference in terminal settling velocities. This concept was popularized by Gaudin, and also appears in a contemporary mineral processing textbook.\(^7\)

However, few quantitative investigations have been carried out on the real effect of an increase in the jiggling frequency on falling during the acceleration period. In addition, a doubt remains as regards applying the acceleration period for particles falling in stationary water (steady motion) to particle motion in jiggling water (unsteady motion). In this study, which focuses on the jiggling movement of a single particle, we confirm the effect of Gaudin’s hypothesis, namely that gravity separation can be improved because high frequency jiggling causes acceleration movement during the early period of falling. We verified the accuracy of our calculation experimentally and we used calculations to investigate the cause and degree of the effect induced by high frequency jiggling.

2. Experimental Methods

2.1 Samples

One particle pairs (one particle in each) consisting of a spherical glass particle (TGK: glass beads #1, 0.991–1.397 mm) and a spherical zirconia particle (Nikkato: YTZ ball φ0.5, average size 0.5 mm), whose settling velocities in stationary water are almost the same, were used as samples. The sample properties are shown in Table 1. The density shown in this table was the average value measured ten times using an electronic densimeter (Mirage Trading: EW-120SG). The particle size and the sphericity are the Heywood diameter and peround, respectively, which were measured by employing image analysis (Mitani: WinRoof) applied to a transmittance microscopic image. The settling velocities of the particles were measured from high-speed camera images (Photoron: FASTCAM Ultima-1024) through a microscope and measured using motion analysis software (Photoron: Movie Ruler).

2.2 Experimental methods

2.2.1 Measuring system

The particle movement in jiggling water was measured with a device that we fabricated shown in Fig. 2. The jiggling movement was generated by a vibration generator (Emic: 512-A), which provides a maximum vibration force of 49 N. A columnar oscillating plate (φ45.7 mm) was attached to the vibration generator and water in a U-tube (inside diameter φ47.5 mm) was oscillated. To realize a large water oscillation amplitude, a tank in the form of a transparent square pillar (13 mm × 13 mm), whose area was smaller than the cross-section of the oscillating plate, was placed on the opposite side to the U-tube. A sine wave or trapezoid-wave signal with an arbitrary frequency generated by a function generator was amplified to an arbitrary amplitude using an amplifier, and the water filling the tank was vertically vibrated through the U-tube from the oscillating plate. A sample particle was placed on a screen with an opening size of 180 μm that was installed inside the tube, and an image of the moving particle in jiggling water was captured by a high-speed camera (Photonor: Fastcam Ultima-1024) through a microscope and measured using motion analysis software (Photonor: Movie Ruler).

2.2.2 Particle movement measurement

800 ml of deionized water in the tank shown in Fig. 2 was de-aired from an air vent port installed at the oscillating plate. The water temperature was measured to check that the temperature had stabilized at 25°C ± 0.5°C. Then, a sample particle was placed on the screen after pre-operating the vibration generator for 15 min. The jiggling condition was adjusted and the particle movement in jiggling water was captured by a high-speed camera. The high-speed camera conditions were as follows: a frame rate of 250–1000 fps, a shutter speed of 1/1000 s, and a resolution of 512 x 1024 pixels (width x height). The capacity of the built-in memory of the high-speed camera permitted a maximum video recording time of about 1 to 4 s under the above conditions. When the particle movement was measured, the vertical centering of the tank was accurately adjusted and only the moving particle, which passed through a 10 mm wide gap in the center, was measured, taking account of the velocity change induced by the inside wall of the tank. The image of the moving particle captured by the high-speed camera was input into a PC, and then the image was measured with motion analysis software. The forms of the water waves were confirmed using the motion analysis software to measure the movement of the gas-liquid interface of the water in the tank, or to measure the movement of tracer particles (glass beads with an average diameter of 2 μm) in the tank, with a high-speed camera. In particular, for trapezoid waves, the wave-

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Table 1 Sample properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (kg/m³)</th>
<th>Particle Size* (μm)</th>
<th>Sphericity</th>
<th>Settling velocity (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>2490</td>
<td>1232.5</td>
<td>0.963</td>
<td>182.3</td>
</tr>
<tr>
<td>Zirconia</td>
<td>6070</td>
<td>492.7</td>
<td>0.966</td>
<td>183.6</td>
</tr>
</tbody>
</table>

*Heywood diameter
form of a function generator was adjusted with wave forming software (NF: 0105) so that the water wave was, as far as possible, an ideal trapezoid wave, because the water wave is generally different from the waveform of a function generator. By measuring the difference between the cross-sectional area of the water at the highest and lowest points using image analysis and dividing this area by the width of the tank (13 mm), the displacement of jigging water can be calculated.

2.2.3 Calculation methods
To calculate the particle orbit in jigging water under ideal conditions and the time a particle takes to reach the screen, we used the following equation called the Tchen or BBO (Basset, Boussinesq, Oseen) equation,\(^8\) which is given by

\[
\frac{\pi}{6} d^3 \rho \rho v \frac{d^2 v}{dt^2} = \frac{\pi}{6} d^3 g (\rho_f - \rho_v) + \frac{C_d}{8} \frac{d^2 \rho t}{dt^2} |u - v| (u - v) + \frac{1}{6} d^3 \rho \left( \frac{du}{dt} - \frac{dv}{dt} \right) + B(t) \tag{1}
\]

where, \(g\) is gravity, \(t\) is time, \(C_d\) is the drag coefficient, \(\chi\) is the added mass coefficient, and \(B(t)\) is the Basset term, which represents the history of the viscous force on the particle.

These differential equations were computed using the Runge-Kutta-Gill method, under initial conditions of \(t = 0\) and \(v = u = 0\), and time steps of \(2 \times 10^{-3} \sim 2 \times 10^{-4}\) s according to the jig frequency by dividing one jigging cycle into 2000. Many equations have been proposed concerning the relation between \(C_d\) and \(Re\), and we used the Haider and Levenspiel equation (eq. (3)), which we also used in our previous report.\(^11\)

\[
C_d = \frac{24}{Re} (1 + 0.1806 Re^{0.6459}) + \frac{0.4251}{1 + \frac{6880.95}{Re}} \tag{3}
\]

The particle orbit with ideal jigging movement and the time until a particle reaches a screen was calculated with the above numeration.

3. Results and Discussion

3.1 Comparison of calculated and experimental results of particle movement

3.1.1 Experimental measurable ranges of jigging frequency and amplitude

The measurable frequency and amplitude ranges of jigging water, in which particles move up and down, are restricted by mechanical and sample conditions. Figure 3 shows the measurable frequency-amplitude ranges of jigging water under the experimental conditions used in this study. The gray area in this figure shows the measurable area that satisfies all conditions (solid line), and the dotted lines indicate the experimental conditions employed in this study. The mechanical upper limit (mechanical limit in Fig. 3) of displacement (twice the amplitude) of the measuring system shown in Fig. 2 is about 110 mm. The greatest restriction to the measurable range is imposed by the border, which shakes as a result of the bubbles from the gas-liquid interface (bubbling limit in Fig. 3). Particle movement cannot be accurately measured under a shaking condition since the orderly jigging condition would be disrupted. In particular, for a trapezoid-wave, which exhibits a large velocity change (the maximum acceleration during one cycle is very high), the measurable range is very small because it is strongly influenced by this shaking effect. In addition, for a trapezoid wave, the measurable range would also be restricted by the limit beyond which the waveform could not maintain a trapezoid shape (the waveform limit in Fig. 3, which is caused by the acceleration limit of the vibration generator). The lifting limit shown in Fig. 3 indicates the minimum limit for lifting the glass and zirconia particles used in this study.

3.1.2 Difference between calculated and experimental orbits of particles

The experimental data and the calculated data obtained from the former BBO equations were compared for both the waveform of jigging water and for the orbits of the glass and zirconia particles in this water. Figures 4(a)(b) shows the experimental and calculated data for jigging water for a sine wave with an amplitude of 5 mm and a frequency of 6 Hz and the orbit of a particle in this water. The experimental data of
water shows the movement of the center point of the gas-liquid interface. The water movement at the gas-liquid interface is not smooth. Even if this fact is ignored, the calculated glass and zirconia displacement is smaller than the experimental values and also the time needed for the particle to rise and reach the screen is different. Therefore, we investigated the reason for the difference between the experimental and calculated particle orbits.

First, we compared the difference between the ideal sine wave motion (calculated value) and the real water motion in our experiment. Figure 5 shows the difference between the experimental and calculated values for the water wave. When the experimental data were compared with the ideal sine wave, this data showed well fitting with the ideal sine wave of 6.5 mm of amplitude (not 5 mm). They showed the same motion for about 70% of one cycle, but the motion for the remaining 30% was different. The secondary wave, which has the same frequency, a smaller amplitude, and a different phase from the water wave, appears when the difference between the two motions is shown in the same figure (differential in Fig. 5). Although the primary factor generating this secondary wave is still unclear, the source was probably the vibration generator and this wave was conveyed from the device stand or the building, because it has the same frequency as the water wave. In any case, these are problems for the experiment, and do not hinder theoretical interpretations.

Next, our attention turned to the structure of the screen. We performed our calculation without considering the presence of a screen, and the calculated water velocity was the same before and after installing the screen. However, the real screen used in this study was a standard sieve made of a wire mesh of plain weave and this wire would be an obstacle to flowing water. The opening size of the screen used in this study is 180 μm; 83 mesh. The area per mesh is (25.4 mm/83)², that is, 0.0937 mm². The open area is 0.18² = 0.0324 mm². Because the opening ratio per area is 0.0324/0.0937 = 0.346, about 2/3 of the area was blocked by the wire of the screen. In other words, the screen acted as an orifice and the water velocity would be increased after passing through the screen. To confirm this phenomenon, the water velocity was measured with a high-speed camera before and after the screen to observe the motion of glass particles (average diameter of 2 μm), which were used as tracer particles. However, it was impossible to observe the surrounding screen section because of the bonding agent used to fix the screen in place. It was observed that the water velocity within 1 mm from the screen was 1.5 to 2 times greater than the average water velocity and that the velocity 2 mm or more from the screen was almost the same as the average water velocity. Because the motion of water that had just passed the screen was complicated and observation near the screen was difficult, we could not measure the attenuation of water velocity with distance from the screen. From the orbits of the water and the sample particles, which were calculated by reflecting their experimental orbits and the experimental change in the water velocity around the screen, it was revealed that the calculated data corresponded to the experimental data when the attenuation of water velocity was as shown in Fig. 6. The water velocity should be 3 times the average velocity at the section where water passes the screen since the closed area is about 2/3. However, if the velocity of the water passing the screen is estimated as 3 times the average velocity, this would constitute an excessive correction, and the calculated data would be far from the experimental data. In this study, twice the average velocity was adopted as the substantial maximum velocity of the water after passing the screen.
From the above results, if we consider the secondary wave and the increasing water velocity induced by the screen, the experimental and calculated orbits would become similar such as those in Fig. 4(c). Since the difference between the experimental and calculated data was caused by our incomplete experimental device, the calculation results without the two correction terms (Fig. 4(b)) can be said to express one particle motion in jigging water accurately if an ideal experimental device had been employed.

3.2 Calculated particle orbits in jigging water

The results described in the previous paragraph showed that the calculation results obtained with eqs. (1), (2), (3) express the particle orbit under ideal conditions well. Therefore, the orbit of one particle and the gravity separation property in jigging water will be investigated based on the results calculated under ideal conditions. As the purpose of this study is the investigation of the relationship between frequencies and the particle orbit in jigging water, the study was carried out by changing the particle orbit with the same amplitude but with different frequencies. The calculated results of the particle orbits for glass and zirconia samples under the measurable conditions given in Fig. 3 are shown in Fig. 7 when using a sine wave, and in Fig. 8 when using a trapezoid wave. Figure 7 shows the orbits of the glass and zirconia particles when the frequency is changed using sine waves with amplitudes of 5, 10 and 20 mm as jigging water motions. The figures are in order of increasing amplitude from the top. The three graphs on the left show the frequency at which both particles were lifted from the screen, when the frequency was increased in 1 Hz steps. In other words, for amplitude-frequency combinations of 5 mm–5 Hz, 10 mm–3 Hz, and 20 mm–1 Hz, neither particle would lift or only one kind of particle would lift. The three graphs on the right show the results of the particle orbits with the bubbling limit shown in Fig. 3 and the three graphs in the center show the results in the middle frequency ranges between those of the graphs on the right and left. The thin black line shows the water wave, the gray line shows the orbit of the glass particles and the thick black line shows the orbit of the zirconia particles. The positions of the particles show the place at which the particle base was in contact with the screen. These figures show the conditions after jigging had already been performed several times, and the time of 0 s in this figure expresses the starting point of the cycles, not the first motion from the stationary state. In every case the glass particle lifts before the zirconia particle. This is because if we compare particles with different specific gravities and the same terminal settling velocities, those with a low specific gravity (B in Fig. 1) are more strongly controlled by the water viscosity when they move from a stationary state. The glass particle always has a higher maximum height, and the difference in the maximum height between the glass and zirconia particles is smaller for higher amplitudes and lower frequencies. Furthermore, the glass particle lifts earlier than the water at 0 s in the three graphs.

Fig. 7 Calculation results of particle orbits of glass and zirconia samples in sine-wave jigging water.
on the right. Although the water height is 0 mm and the water velocity is 0 m/s at a time of 0 s, the water has already experienced upward acceleration. In particular, under the three conditions shown on the right in the figure, which show the measurement limits, this phenomenon occurs because the water acceleration, namely the pressure gradient force that is sufficient to lift a glass particle, works on the particle.

Figure 8 shows the orbits of glass and zirconia particles in the jigging water when the frequency is changed to trapezoid waves with amplitudes of 5, 10, and 20 mm. The trapezoid wave of the water was patterned so that the maximum acceleration of the jigging water would not exceed the maximum acceleration realized by the device shown in Fig. 2. The three graphs on the left show the results at frequencies when both particles first lifted from the screen when increasing the frequency in 1 Hz steps as performed with the sine wave. However, because the measurable range with the trapezoid wave is small, as shown in Fig. 3, it is impossible to keep the waveform at an amplitude of 5 or 10 mm and over 4 Hz (waveform limit). The measurement is also impossible at an amplitude of 20 mm and over 3 Hz because bubbles are generated (bubbling limit). Although water is still at around 0.3 seconds for an amplitude of 10 mm and a frequency of 3 Hz, the glass particle lifts slightly. This is because the upward acceleration (pressure gradient force) works on a motionless particle on a screen when the water velocity is zero at \( v_{\text{min}} \) as a result of the upward acceleration, after the downward water velocity reaches its maximum value at \( v_{\text{max}} \). The reason for only the glass particle rising here is as mentioned above. The glass particle lifts before the zirconia particle in every case, as found with the sine wave. The maximum height of a glass particle is always greater than that of a zirconia particle. However, this difference would be smaller with an increase in amplitude, and these particles move in almost the same orbits at an amplitude of 20 mm and at frequencies of 1 and 2 Hz.

### 3.3 Calculated size ratio when particles reach screen simultaneously

The results in the previous paragraph showed that the zirconia particles always land earlier on the screen than the glass particles after the particles have been lifted by water jigging. So, we compared the cumulative “difference in time taken to reach the screen” (hereafter abbreviated to “DTRS”), which were calculated by multiplying the frequency (Hz) with the DTRS of the two particles under each jigging condition. This cumulative value is the time during which glass particles still exist in the water one second after a zirconia particle has landed on the screen. As these samples are particles falling with the same velocity in stationary water, a long DTRS means that there is a stronger tendency for separation according to the difference in the specific gravity of the particles. Figure 9(a) shows the relationship between the frequency and the cumulative DTRS, when the jigging water is a sine wave or a trapezoid wave under the amplitude conditions in the measurable area. The cumulative
DTRS for any waveforms and amplitudes extended with increases in frequency, and the accuracy of the gravity separation between the glass and zirconia particles was improved. Figure 9(b) shows the primary regression line of the least-squares method drawn for all the plots shown in Fig. 9(a). Strictly speaking, the tendencies are different under each condition. However, it is recognized that the relation between the frequency and the cumulative DTRS shows good linearity ($R^2 = 0.949$) as a whole, regardless of the use of a sine wave or a trapezoid wave. From the regression line equation, for the water jigging of the samples used in this study, the time differential as regards reaching the screen between the glass and zirconia particles in one second is extended by 0.016 s with an increase of 1 Hz. Here, the plot at a frequency of 6 Hz and an amplitude of 5 mm shown in Fig. 7 was omitted from this distribution, since the zirconia particles hardly moved.

Figure 10 shows the DTRS for both particles under each jigging condition, namely the relations of frequency, amplitude, and the maximum acceleration of the jigging water are shown for the motion of one cycle before multiplying it by the frequency. Innumerable wave patterns can be assumed geometrically when fixing the frequency and amplitude because the waveform of the trapezoid wave does not only depend on the frequency and amplitude. In this study, we only investigated the trapezoid wave shown in Fig. 8, which is within the measurable range in Fig. 3. Figure 10(a) shows the relation with frequency. Although the plots of a trapezoid wave, which cannot be compared with a sine wave under the same condition, have very different patterns from the others, the other plots remain largely linear even when compared with the difference in the time needed to reach the screen in one cycle. It must be specially mentioned that the DTRS is extended with increases in frequency although the time of one period decreases. Figure 10(b) shows the relation with the amplitude. Although the DTRS tends to become smaller with increased amplitude, the plots are distributed widely as a whole because the degree depends on the frequency.

Figure 10(c) shows the relation with the maximum acceleration of jigging water. In the relation with the maximum acceleration, a trapezoid wave cannot be an object of comparison. However, when we observe the tendency for the sine wave, these plots are distributed widely, but are similar to the relations with the frequency, since the DTRS is increased with increases in acceleration.

The particle motion in jigging water is expressed as eqs. (1)~(3). The forces that participate in the jigging motion of water are drag force and the pressure gradient force of water. The former is a function of the relative velocity between water and a particle and the latter is a function of water acceleration. Here we consider the motion of glass and zirconia particles, which have the same terminal settling velocity in stationary water. The degree of the following of jigging motion of water is always better for a glass particle than a zirconia particle as shown in Figs. 7 and 8. Therefore, the relative velocity of a glass particle to water is always smaller than that of a zirconia particle. This means that water imposes a relatively strong drag force on a zirconia particle and a relatively strong pressure gradient force of water on a glass particle. As an example, Fig. 11 shows the change in each force element received until the glass and zirconia particles reach the screen at a frequency of 6 Hz and an...
amplitude of 10 mm. The common features are that the gravity force does not change over time and the Basset history force is weaker than the other forces. Next, we discuss the influence of the drag force and the pressure gradient force. The total force that a glass particle receives corresponds to the change in the pressure gradient force in Fig. 11(a), and this means that a glass particle is influenced more strongly by pressure gradient force than by drag force. In contrast, the total force that a zirconia particle receives corresponds to the change in the drag force in Fig. 11(b), and this does not greatly influence the change in the pressure gradient force. In other words, the motion of a glass particle strongly depends on pressure gradient force, which is a function of water acceleration, and the motion of a zirconia particle strongly depends on drag force, which is a function of water velocity. When the focus is on the sine-wave motion of jigging water, water velocity is described as $A \sin \omega t$ and water acceleration is described as $A \omega^2 \sin \omega t$. Here, $A$ is amplitude and $\omega$ equals $2\pi f$ ($f$: frequency). No additional difference is generated in the motion between the glass and zirconia particles since drag force and pressure gradient force exhibit a similar tendency to increase because the water velocity and acceleration increase simultaneously when the amplitude is increased. On the other hand, when the frequency is increased, the increase in the pressure gradient force tends to become larger than that of the drag force because the water velocity and acceleration increase in proportion to the frequency and the water acceleration increases in proportion to the square of the frequency. Therefore, the change in the motion of a glass particle is larger than that of a zirconia particle and an additional difference as regards the motion between the two particles would be generated. The difference between the equation for the falling motion of a particle in stationary water and the oscillating motion of a particle in jigging water is the presence of a pressure gradient force (water acceleration). Particles with the same terminal settling velocity in stationary water move differently in jigging water because the pressure gradient force makes the increase in the water acceleration greater than the water velocity. The $DTRS$ strongly depends on frequency and tends to increase with increases in frequency in Figs. 9 and 10. This is simply because the increase in frequency corresponds to the way that the pressure gradient force is applied thus making the increase in the water acceleration larger than that of the water velocity.

The results reported in this study show the characteristics of the improvement in the accuracy of gravity separation with increases of frequency in jigging water, which is the same as Gaudin’s hypothesis. However, the reason for this was different from that behind Gaudin’s hypothesis. The real reason is not directly related the difference in the acceleration motion during the early period of particles falling in stationary water, where the presence of a pressure gradient force is not estimated. We performed the following calculations to compare the degree to which the effects in this study and those of Gaudin’s hypothesis are valid. First, we assumed that the falling time was half of the jigging cycle to allow us to estimate the degree of improvement in gravity separation accuracy, which could be estimated from Gaudin’s hypothesis, for the glass and zirconia particles used in this study. As regards the glass particles, we calculated the time taken by a stationary zirconia particle to fall the distance that a stationary glass particle falls in stationary water for this assumed falling time and we calculated the difference between the two falling times as the $DTRS$. As regards the zirconia particles, we calculated the $DTRS$ by reversing the glass and zirconia particles in the above case. The cumulative $DTRS$ values were finally compared by multiplying the frequency by the $DTRS$. Figure 12 compares the calculated data obtained in this study and shown in Fig. 9 with the effect of Gaudin’s hypothesis estimated by the calculation shown above. Unexpectedly, the two sets of data agree very well for the samples used in this study. The calculated results in this study are almost replicated in reality, and if this is considered, it may be said that Gaudin’s hypothesis provides a superior indication that does not seriously contradict the real effect for times when it was difficult to calculate a differential equation accurately.
4. Conclusion

To verify the real effect of Gaudin’s hypothesis, which says that jigging water with a high frequency improves the accuracy of gravity separation, we investigated the degree of this effect and the reason for it by focusing on one particle motion in jigging water. The results can be summarized as follows.

(1) When one particle motion in jigging water was calculated using the BBO equation and compared with the experimental results, the calculated data corresponded to the experimental data well if the problems with the device were eliminated such as the effect of the secondary wave, which is not related to the jigging motion, and the effect of the increase in the velocity of water when it passes a screen. These results showed that the calculation methods described in this paper express the ideal particle motion in jigging water well.

(2) The calculated results of the orbits of glass and zirconia particles in jigging water with several waveforms, frequencies, and amplitudes, showed that glass particles lifts first and reach a high position, and zirconia particles always reach the screen later except when both particles move almost the same amount. This is because a particle with a low specific gravity is strongly controlled by water viscosity when particles with different specific gravities and the same terminal settling velocity start to move from stationary states.

(3) The reason for the improvement in the accuracy of gravity separation by the high frequency water jigging for glass and zirconia particles was estimated by using an index called the cumulative $DTRS$. These results clearly showed that these phenomena occurred because an increase in frequency corresponds to the way that the pressure gradient force is imposed thus making the increase in the water acceleration greater than that of the water velocity. However, it was concluded that although the reason is incorrect, Gaudin’s hypothesis reflects the actual phenomenon since the effect calculated in this study almost agrees with estimations obtained using the classical hypothesis approach developed by Gaudin.

REFERENCES


Appendix: Nomenclature

$v_{m1}$, $v_{m2}$: terminal settling velocities of particles
$d$: particle diameter
$\rho_p$: particle density
$\rho_w$: water density
$v$: particle velocity
$u$: water velocity
$g$: gravity
$t$: time
$C_d$: drag coefficient
$\chi$: added mass coefficient
$B(t)$: Basset term
$\mu_w$: water viscosity
$Re$: Reynolds number
$DTRS$: difference in time taken to reach the screen
$A$: amplitude
$\omega$: angular velocity
$f$: frequency