Constitutive Relation of Casting Aluminum Alloy A101 Involving Void Evolution

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Casting aluminum alloys are highly heterogeneous materials with different types of voids that govern the mechanical behavior of the material. In this paper, based on the analysis of a cylindrical void model and the assumption of matrix incompressibility, the void evolution of a casting aluminum alloy is derived. Through the analysis of micro-velocity and the strain fields of the cylindrical void model, an endochronic constitutive equation involving void evolution is obtained for cast aluminum alloys. The corresponding finite element procedure is developed and applied to the analysis of the mechanical behavior and the porosity of casting aluminum alloy A101. The computed results show satisfactory agreement with experimental data.

(Received June 21, 2005; Accepted October 4, 2005; Published December 15, 2005)

Keywords: casting aluminum alloy, cylindrical void, void evolution, elastoplasticity

1. Introduction

Casting aluminum alloys are attracting increasing attention in transportation vehicle applications due to their desirable characteristics in reducing vehicle weight and amenability to recycling. However, because casting aluminum alloys contract when solidified, and because hydrogen dissolves easily in the melted metal, the materials inevitably contain a certain amount of voids. These voids would grow when the materials bear external loading, which may degrade the ductility of the materials and cause fracture of the materials. The constitutive relation of casting aluminum alloys should involve the effect of void growth on the mechanical properties of the material and the analysis method for the ductile fracture of porous material can be used in the analysis of the effect of void growth.

It is known that voids usually distribute randomly in a porous material. In order to study the effects of the growth of voids on the mechanical properties of porous material, a viable method used frequently for the ductile fracture of porous material is to research a simple representative cell of the material. In such analysis, Gurson made a pioneering contribution. He assumed that the void-matrix aggregate of a porous ductile media is a single void in a rigid-plastic cell, and the void volume fraction (porosity) of the cell equals that of the aggregate. Then he adopted an upper bound condition to obtain the constitutive model for solids containing circular-cylindrical or spherical voids. Licht and Suquet investigated the growth of a cylindrical cavity in a finite shell of a nonlinear power law viscous material and obtained a closed solution. Reusch et al. extended the Gurson’ model to the case of isotropic ductile damage and crack growth. Perrin et al. presented an analytical and numerical study of accelerated void growth in porous ductile solids. Taylor et al. studied the effect of void size and spacing on the ductility and flow stress of viscoplastic materials. Tvergaard and Niordson analyzed the effects of nonlocal plasticity on the interactions of voids of different sizes based on an axisymmetric unit cell model with special boundary conditions.

In this paper, void growth relations are derived for casting aluminum alloy A101 based on the analysis to a cylindrical void-cell model presented by Gurson. Making use of the Gurson’s void-cell model and the assumption of homogenization, the elastoplastic constitutive description was obtained for casting aluminum alloy. A corresponding finite element implementation was developed and used to analyze the stress and the porosity distribution of the material subjected to tensile loading. The obtained results are in satisfactory agreement with experimental data.

2. Cylindrical Void Model and Void Growth Equation

A representative material element (RME) of casting aluminum alloys is shown in Fig. 1, in which many microvoids stochastically distribute. The matrix of the REM is assumed homogeneous and incompressible, and the RME is large enough to be statistically representative of the properties of the material. Because cylindrical voids are often found in cast aluminum alloys which might result from cylindrical inclusions (e.g., silicon grain), it is supposed that the voids in the material element have cylindrical shape. The material element including the cylindrical voids can be simplified as a cylindrical void-cell model (Fig. 2). The initial and current inner and outer radii of the void cell are \(a_0, b_0\) and \(a, b\), respectively; the initial and current radii of an arbitrary point in the matrix of the cell are \(R\) and \(r\), respectively; and the initial and current lengths of the cell are \(l_0\) and \(l\) respectively. From Fig. 2, we may write

![Fig. 1 Material element with voids.](image)
Combining eqs. (2), (3) and (4), we obtain
\[ V = \frac{a^2}{b^2}, \quad f = \frac{a^2}{b^2}. \] (3a,b)

Introducing logarithmic strain,
\[ \frac{b}{b_0} = \exp(-\varepsilon_{11}), \quad \frac{l}{l_0} = \exp(-\varepsilon_{33}). \] (4a,b)

Combining eqs. (2), (3) and (4), we obtain
\[ f = 1 - (1 - f_0) \exp(-\varepsilon_{48}). \] (5)

Equation (5) is the evolution of porosity, which shows that the current void volume fraction is related to the initial void volume fraction and volumetric strain. From eq. (5) it can be seen that the increment of void volume fraction develops with the volumetric strain for each initial void volume fraction.

### 3. Constitutive Relation

The macroscopic stress and strain tensors in the void cell are denoted by \( \Sigma_{ij} \) and \( E_{ij} \), and the corresponding microscopic stress and strain tensors are \( \sigma_{ij} \) and \( \varepsilon_{ij} \), respectively. The volumes of the matrix and the void of the cell are denoted by \( V_m \) and \( V_v \), respectively, and the volume of the cell is defined as \( V = V_v + V_m \). The volume dilatation is completely due to the growth of void. The macroscopic deformation rate of the cylindrical void cell can be defined in terms of the velocity field at the outer surface of the cell,\(^3\) i.e.,
\[ \dot{E}_{ij} = \frac{1}{V} \int_S \frac{1}{2} (v_i n_j + v_j n_i) dS, \] (6)

where \( v_i \) is the microscopic velocity field, \( S \) is the outer surface of the cell, \( n_j \) is the unit outward normal on \( S \). For the macroscopic stress field and strain-rate, the following boundary conditions should be satisfied under Cartesian coordinates:
\[ p_i = \sigma_{ij} n_j = \Sigma_{ij} n_j \quad (r = b) \] (7a)
\[ p_i = \sigma_{ij} n_j = 0 \quad (r = a) \] (7b)

where \( p_i \) is the external pressure applied on the outer surface of the cell, and \( n_j \) is the local orthogonal principal axis direction. From Fig. 2 and eq. (8), the boundary velocity field in the cylindrical coordinate system \( (r, \theta, z) \) can be obtained as
\[ v_1|_b = E_{11} b \cos \theta, \quad v_2|_b = E_{22} b \sin \theta, \quad v_3|_b = E_{33} x_3, \] (9)

and the microscopic strain rate \( \dot{\varepsilon}_{ij} \) in the matrix can be expressed as
\[ \dot{\varepsilon}_r = d v_r / d r, \quad \dot{\varepsilon}_\theta = v_r / r, \quad \dot{\varepsilon}_z = d v_3 / d x_3, \] (10)

where \( v_r \) and \( v_3 \) are the components of the velocity field \( v \). Considering an axisymmetric deformation, one has
\[ E_{11} = E_{22} \neq 0, \quad E_{33} = w E_{11}, \quad E_{ij} = 0 \quad (i \neq j), \] (11)

where the parameter \( w \) is the strain constraint function which incorporates the effect of stress triaxiality of void growth; it lies in the range \( 0 \leq w \leq 2 \).\(^8\) Making use of these relations and invoking the condition of matrix incompressibility, i.e.,
\[ \dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{33} = 0, \] (12)

we obtain
\[ \dot{\varepsilon}_r = \frac{1}{2} \left( \frac{w - b^2}{r^2} (2 - w) \right) \dot{E}_{11}, \] (13a,b,c)
\[ \dot{\varepsilon}_z = \frac{1}{2} \left( \frac{w + b^2}{r^2} (2 - w) \right) \dot{E}_{11}, \quad \dot{\varepsilon}_z = w \dot{E}_{11}. \] (14a,b)

The endochronic theory of plasticity introduced by Valanis\(^9\) provides an alternative approach to classical plasticity for describing the inelastic behavior of history-dependent materials. A key point of the theory is that the irreversible deformation history is not measured with real (Newtonian) time but rather with material time defined in term of irreversible deformation of materials.\(^9\) This theory provides a unified approach for describing the elastic-plastic behavior of many materials that do not have distinct yield surface (or yield point). It does not require the notion of yield surface or the specification of a loading function to distinguish loading from unloading. Because of these merits, endochronic theory is used to explore the constitutive equation for casting aluminum alloys that do not distinctly exhibit yield. The endochronic constitutive equation proposed by Valanis\(^9\) is written as
\[ s = \int_0^\sigma \rho(z - z') \frac{d e^p}{d z'} d z' \text{ tr}(d \sigma) = 3 K t r(d e), \] (14a,b)
\[ \rho(z) = \sum_{r=1}^a C_r e^{-a_r z}, \quad d z = d \xi / F(\xi), \] (15a,b)
\[ d \xi = \| d e \|, \quad d e^p = d e - d s / 2 G. \] (16a,b)

Here, \( \sigma \) and \( s \) denote respectively the tensors of microscopic stress and its deviatoric component, \( e \) and \( e^p \) are the tensors of macroscopic strain, its deviatoric component and plastic component, and \( \rho(z) \) is a weakly singular kernel function that satisfies the condition \( \rho(0) = \infty \). Moreover, \( G \) and \( K \) are respectively the shear and bulk moduli, \( C_r \) and \( a_r \) are material constants, \( F(\xi) \) is the hardening function related to irreversible deformation, where \( \xi \) and \( z \) are the intrinsic time.
measure and the intrinsic time scale, respectively, and \( \| A \| \) denotes the Euclidean norm of tensor \( A \). Substituting eq. (13) into eq. (16a), the increment of intrinsic time measure can be expressed as

\[
d\zeta = \sqrt{b^2(2 - w)^2 + 3w^2r^4dE_{11}}/\sqrt{2r^2}.
\]  

(17)

Considering that the intrinsic time scale is not only related to the irreversible deformation of material but also to the evolution of void volume fraction, a new intrinsic time is defined as

\[
d\tilde{\zeta} = d\zeta/F(\zeta)\eta(f),
\]  

(18)

where \( F(\zeta) \) is the hardening function which reflects the hardening behavior of material under plastic deformation, \( \eta(f) \) is the softening function which reflects the softening behavior of material owing to the void growth in the course of plastic deformation. For simplicity, \( F(\zeta) \) and \( \eta(f) \) are given simple forms that neglect strain rate effects, i.e.,

\[
F(\zeta) = 1 + \beta_1\zeta^\gamma_1, \quad \eta(f) = 1 + \beta_2f^\gamma_2,
\]  

(19a,b)

where \( \beta_1, \beta_2, \gamma_1, \) and \( \gamma_2 \) are material constants, which were determined by the former and latter sections of a tensile test curve, respectively. Substituting \( d\varepsilon^B/d\tilde{\zeta} \) in eq. (14) with \( \Delta\varepsilon^p/\Delta\zeta \) and making use of the mean value theorem,\(^{10}\) an incremental form of the microscopic constitutive equation can be expressed as

\[
\Delta s = \sum_{r=1}^{3}(1 - \exp(-\alpha_r\Delta\zeta))\left( \frac{C_r}{\alpha_r} \frac{\Delta\varepsilon^p}{\Delta\zeta} - s'(\hat{\zeta}_{rn}) \right),
\]  

(20)

where \( \hat{\zeta}_r \) is the intrinsic time scale after \( r \)th increments of loading, \( \hat{s}'(\hat{\zeta}_r) \) represents the \( r \)th component of deviatoric stress. Ignoring the effect of voids on the constitutive equation \( (f = 0) \), it can be proved that the constitutive eq. (20) reduces to the incremental form of the nonclassical constitutive equation given by Peng and Fan.\(^{11}\) Moreover, Chaboche’s constitutive law for back stress can also be obtained as a special case when \( \eta(f) = 1 \) and \( F(\zeta) \) is constant.

Because the matrix is assumed homogeneous, the homogenization principle can be used in the transition between microscopic and macroscopic quantities.\(^{21}\) Letting \( \Phi \) and \( \phi \) be the macroscopic and the microscopic potential functions, respectively, the macroscopic stresses can be expressed as

\[
\Sigma_{ij} = \frac{\partial \Phi}{\partial E_{ij}} = \frac{1}{V} \int_{V_n} \frac{\partial \phi}{\partial E_{ij}} dV = \frac{1}{V} \int_{V_n} s_{ij} \frac{\partial \varepsilon_{11}}{\partial E_{ij}} dV.
\]  

(21)

Substituting eqs. (13) and (20) into eq. (21), The macroscopic constitutive equation for casting aluminum alloy can be obtained.

### 4. Application and Verification

The material used is casting aluminum alloy A101. The geometry of the cylindrical specimens is 200 mm in length, and 10 mm in diameter. The geometry of the notched cylindrical specimen is shown in Fig. 3. The eight-node isoparametric element with \( 2 \times 2 \) Gaussian points is adopted in the finite element algorithm. The axial displacements were given at the end of the specimens with the incremental step 0.02 mm, and no radial constraint is applied to the surface of the specimens. The experimental data were obtained from tensile tests, which were conducted on an Instron 1342 servohydraulic material testing system. Figure 4 shows the stress–strain relation obtained by both computation and experiment for the cylindrical tensile specimen. Satisfactory agreement of the stress–strain relationship between computation and experiment is shown. The results also show that the material has no distinct yield point. In order to experimentally determine the relationship between the porosity and deformation, a set of cylindrical specimens was loaded to different stress level and then unloaded (see Fig. 4). After these specimens with different deformations were cut along the radius at their minimum cross section and polished, a photo-interpreter was used to quantify the porosity on the minimum cross sections of these specimens. Figure 5 compares the relationships of the porosity and plastic deformation obtained by experiment and calculation. From Fig. 5 it is evident that the computed results agree reasonably with the experimental results. The porosity increases with the increase of the plastic deformation, approximately following an exponential law. Figure 6 shows a metallograph at the fracture section of the specimen obtained with a scanning electron microscope, from which many dimples can be observed. These dimples are indicative of ductile fracture of the material. A notched cylindrical tensile specimen was loaded to 30 kN followed by unloading. The distribution of the porosity along the radius of the specimen was recorded. Figures 7 and 8 show the distributions of the axial stress and porosity. In Figs. 7 and 8, the coordinate 0 denotes the surface of the smallest cross section of the specimen, 5 mm corresponds to the center of the cross section of the specimen, and \( a \) denotes the distance
from the surface. From Fig. 7 it can be seen that the axial stress reaches a maximum value at the root of the notch and decreases rapidly toward the center of the specimen. The larger the initial void volume fraction is, the larger the axial stress is. From Fig. 8 it can be found that the porosity also occurs a maximum value near the root of the notch of the specimen and the computed and the experimental results also show reasonably agreement.

5. Conclusions

Based on a cylindrical void-cell model and the assumption of matrix incompressibility, the void evolution for a casting aluminum alloy was derived. Then, through the analysis of the velocity field and the boundary conditions of the cylindrical void-cell model, the strain rate field of the void-cell was obtained. Defining a new intrinsic time that includes a softening function related to current void volume fraction, the constitutive equation for casting aluminum alloy was obtained. The corresponding numerical algorithm and the finite element implementation were developed and applied to the analysis of the stress–strain and porosity-deformation relationships of two kinds of cylindrical tensile specimens. It was found that the computed and the experimental results are in satisfactory agreement.

Acknowledgement

The financial supports from Natural Science Foundations of China (Grant No. 10272120 and 10572157) and Chongqing (Grant No. 2005BB4119) are fully acknowledged.

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