Theoretical Analysis on Crystal Alignment of Feeble Magnetic Materials under High Magnetic Field

Cunyou Wu¹, Shuqin Li², Kensuke Sassa¹, Yasumasa Chino³, Kazutoshi Hattori¹ and Shigeo Asai¹

¹Dept. of Materials Processing Engineering, Graduate School of Engineering, Nagoya University, Nagoya 464-8603, Japan
²Institute of Chemistry, Chinese Academy of Science, Zhong Guan Cun Beiyijie No. 2 Hai Ding region, Beijing 10080, China
³National Institute of Advanced Industrial Science and Technology (AIST), 2266 Anagahora, Nagoya 463-8560, Japan

Since superconducting magnet was developed, high magnetic field has been used as one of the effective ways to get textured structures by aligning crystals in materials. It is well known that the high magnetic field can align crystals having magnetic anisotropy. However, effects of Brownian motion of crystals in liquid medium and gravity force on the crystal alignment are not well known. In this study, it has been found that there is a size range of crystal particles in which the crystals can be aligned by high magnetic field under actions of Brownian motion of crystals and gravity force. Moreover, by taking account of these factors, theoretical analysis of crystal alignment under high magnetic field has been carried out to elucidate the phenomena of the crystal alignment in both feeble magnetic materials having well or poor electric conductivity.

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1. Introduction

Control of structures in ceramics or metals improves their physical and mechanical properties. Ceramics having textured structures can be produced by such processes as templated or seed grain growth,¹²³ hot forging or pressing³⁴ and tape casting.⁵ On the other hand, since technologies of superconducting magnets were developed, high magnetic field has become one of the effective ways to get the textured structures by controlling crystal alignment.⁶

Recently, a lot of new phenomena and functions of materials have been found in the fields of science and engineering relating to the high magnetic field. Most of these phenomena and functions are caused by magnetization force. The magnetization force resulted from the high magnetic field has two kinds of action. One is to attract ferromagnetic and paramagnetic materials or to repel diamagnetic ones. The other is to rotate a unit cell of material (like a crystal) around a certain crystal axis to align the crystal to the direction of magnetic field. The former force is usable mainly for magnetic separation, magnetic levitations⁷ or measurements of magnetic susceptibility of materials. The latter is applicable for alignment of crystals or formation of textured structures in materials by using difference in magnetic susceptibility among crystal axes or in shape. In fact, the high magnetic field can fabricate textured microstructures even for feeble magnetic materials, when the anisotropic magnetic energy on the magnetization is strong enough to align crystal particles.⁸⁻¹⁵ Many materials have the magnetic anisotropy of crystals, namely magnetic susceptibility is different in each crystal axis and thus the crystals can be rotated to align to a preferred direction. Recently, the application of high magnetic field for control of structures of materials is recognized as one of the useful technologies in materials processing.

Theoretical analyses of crystal alignment in feeble magnetic materials, which have very low magnetic susceptibility and thus very weak magnetization, have been reported for the case where materials have well electric conductivity¹⁶ or poor electric conductivity.¹⁷ However, these researches didn’t consider effects of crystal size on the alignment. On the other hand, some researchers have recently found that highly orientated textures in ceramics having feeble magnetic susceptibility can be manufactured by imposition of high magnetic field in a slip casting of suspended ceramic powders and followed with sintering.¹⁸⁻¹⁹ However, until now, the theoretical analysis on the rotation and alignment of ceramic crystals under high magnetic field has never been reported. In this study, the crystal alignment under high magnetic field has been theoretically analyzed by taking account of crystal size, Brownian motion and gravity force for feeble magnetic crystals having well or poor electric conductivity in spherical or rod-like shape.

2. Crystal Alignment by Imposition of High Magnetic Field

When non-magnetic substance is magnetized in magnetic field, the energy for magnetization of the substance is given by eq. (1).

\[ U = -\frac{1}{\mu_0} \int B \cdot dB_{\text{in}}, \quad (1) \]

where \( M \) is magnetization, \( B \) and \( B_{\text{in}} \) the imposed magnetic flux density and the magnetic flux density in the substance, respectively, and \( \mu_0 \) magnetic permeability in vacuum (4π × 10⁻⁷ [H/m]). The principle of crystal alignment using magnetic field is that a magnetic torque, which is generated in a crystal by interaction between the magnetization of the crystal and the imposed magnetic field, rotates the crystal so as to take a stable crystal orientation and to decrease the magnetization energy.²⁰⁻²¹

Let us discuss the case where a material has a crystal structure with magnetic anisotropy, i.e. the magnetic susceptibility is different in each crystal direction. For example, the magnetic susceptibilities in a hexagonal crystal are different along a- or b-axis and c-axis. The value of the magnetization energy given by eq. (1), which depends on the 

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magnetic susceptibility of each crystal axis and a crystal shape, determines a preferred crystal orientation.

\[ U = -\frac{\chi}{2\mu_0(1 + N\chi)} B^2, \]  

(2)

where \( N \) is demagnetization factor and \( \chi \) magnetic susceptibility. When \( U_c < U_{c,b} \), c-axis of crystal is oriented to the preferred direction, which is in parallel to the direction of magnetic field. On the contrary, when \( U_c > U_{c,b} \), a- or b-axis of crystal is oriented to the preferred direction in parallel to the direction of magnetic field. When crystals are set in a magnetic field, they tend to align to the preferred crystal orientation.

Figure 1 schematically shows the crystal alignment due to high magnetic field. When \( \chi_c > \chi_{c,b} \), namely \( U_c < U_{c,b} \), where \( \chi_c \) and \( \chi_{c,b} \) are magnetic susceptibilities of c-axis and a- or b-axis, respectively, c-axis of crystals is oriented to the preferred direction in parallel to the direction of magnetic field. Therefore, it is relatively easy to get highly textured structure in which c-axis of crystals is aligned to one direction. On the contrary, when \( \chi_c < \chi_{c,b} \), i.e., \( U_c > U_{c,b} \), a- or b-axis of crystals is oriented to the preferred direction, which is in parallel to the direction of magnetic field, and thus c-axis of crystal is randomly oriented to all the directions in perpendicular to the direction of imposed magnetic field.

3. Range of Effective Particle Size for Crystal Alignment under High Magnetic Field

In this chapter, the crystal alignment due to magnetization force is theoretically analyzed to elucidate the size of crystal particles effective for the crystal alignment. It is well known that Brownian motion is more active for small particles while the motion due to the gravity force is more active for large ones. Therefore, when considering these two factors, it is necessary to classify the crystal size for crystal alignment under high magnetic field.

3.1 Maximum size of crystal particle effective for the alignment under high magnetic field

For a large crystal particle, effect of gravity force on the crystal alignment must be taken into consideration, while Brownian motion of crystals in liquid medium can be ignored.

The terminal settling velocity of a particle in liquid medium due to the gravity force is given by eq. (3).

\[ v_T = \frac{2g(\rho_p - \rho_l)r^2}{9\eta}, \]  

(3)

where \( g \) is acceleration of gravity, \( r \) radius of particle, \( \eta \) viscosity of liquid medium, \( \rho_l \) density of the medium, and \( \rho_p \) density of particle.

For a spherical particle, the time needed for the particle to align due to the magnetization force, \( \tau_m \), can be evaluated by eq. (4), which will be derived in the section 4.1.

\[ \tau_m = \frac{6\eta\mu_0}{\Delta\chi B^2}, \]  

(4)

where \( \Delta\chi \) is difference of magnetic susceptibilities between crystal axes.

The rotation of crystals due to the magnetization force must finish before the particle sediments to the bottom of a vessel. This condition can be expressed as eq. (5) with a falling length \( L \) for the sedimentation of particles.

\[ t_m < L/v_T \]  

(5)

Substituting eqs. (3) and (4) into eq. (5) yields the maximum radius of a crystal particle under which the particle can rotate and resultantly aligns according to the direction of magnetic field, as shown in eq. (6).

\[ r < \sqrt{\frac{3\Delta\chi B^2 L}{4\mu_0 g(\rho_p - \rho_l)}} = r_{\text{max}} \]  

(6)

When the crystal size is larger than \( r_{\text{max}} \), crystal alignment cannot be finished, while the crystal size is smaller than \( r_{\text{max}} \), crystal alignment is obtainable.

3.2 Critical size of particle for the Brownian motion under gravity force

For a small spherical particle suspended in a fluid, the Brownian motion can take place. The displacement due to the Brownian motion, namely, the distance of the movement of particle in the fluid, can be expressed by eq. (7). \(^{22}\)

\[ x(t) = \sqrt{\frac{2DkT}{\zeta}} \left[ 1 - e^{-t/\tau_R} \right], \]  

(7)

where \( \zeta = 6\pi\eta r \) is the friction coefficient, and \( \tau_R = m/\zeta = 2\rho_p r^2/9\eta \) the relaxation time, which means a time until the particle reaches the equilibrium state after disturbance. Here, \( D \) is number of spatial dimensions, \( k \) Boltzmann constant \((1.38 \times 10^{-23} \text{J/K})\), \( T \) temperature of fluid and \( t \) time.

When the average velocity of a particle caused by the Brownian motion in fluid is larger than the terminal velocity of the settling that caused by the gravity force, then the gravity force may be ignored in the theoretical analysis of crystal rotation in fluid. This condition is expressed by eq. (8).

\[ x(t_R)/\tau_R > v_T \]  

(8)

In eq. (8), when only the direction of gravity is considered, the number of spatial dimension \( D \) is equal to 1. Substituting eq. (7) into eq. (8) yields eq. (9) as the equation of a critical radius of particle under which the Brownian motion has to be
taken into consideration for the crystal alignment due to high magnetic field but the gravity force can be ignored.

\[
r < \sqrt{\frac{243kT\eta^2}{8\pi\rho\varepsilon_0(\rho_p - \rho)}} \equiv r_{\text{cri}}
\]  
(9)

3.3 Minimum size of crystal particle effective for the alignment under high magnetic field

Rotation of crystal particle can take place by the Brownian motion of the particle in fluid. The relaxation time of the Rotational Brownian motion is expressed by eq. (10).

\[
\tau_B = \frac{3\eta V}{kT},
\]  
(10)

where \( V \) is volume of particle.

It is known that Brownian motion is more active when the smaller the particle is. In order to effectively align the particle under high magnetic field, the particle must be large enough to overcome the Brownian motion. The time for the alignment \( \tau_m \) must be shorter than the relaxation time of the Brownian motion \( \tau_B \). This condition is expressed as eq. (11).

\[
\tau_m < \tau_B
\]  
(11)

Substituting eqs. (4) and (10) into eq. (11) yields eq. (12).

\[
r > \sqrt{\frac{3kT\mu_0}{2\pi\Delta\chi B^2}} \equiv r_{\text{min}}
\]  
(12)

Equation (12) expresses the minimum size of crystal particle over which the particle can be aligned under high magnetic field, regardless of the Brownian motion of the particle.

3.4 Range of effective particle size for crystal alignment under high magnetic field

According to the theoretical consideration mentioned above, the range of effective particle size for crystal alignment under high magnetic field is schematically shown in Fig. 2. By applying a high magnetic field, which is strong enough to induce the magnetization force, the range of effective crystal particle size for the alignment can further be classified as follows:

1) \( r_{\text{min}} < r < r_{\text{cri}} \):

In this range, the crystal alignment due to high magnetic field can take place under the condition that the velocity due to the Brownian motion is faster than the settling velocity of particle due to gravity force. That is, the gravity force can be ignored for the crystal alignment in this range.

2) \( r_{\text{cri}} < r < r_{\text{max}} \):

In this range, the Brownian motion of particle can be ignored for crystal alignment, but both of the gravity force and magnetization force should be taken into consideration.

4. Rotation of Spherical Crystal Particle

For particles whose radii are in the range of \( r_{\text{min}} < r < r_{\text{cri}} \), the Brownian motion will not affect the crystal alignment, although it overcomes the gravity motion. In this paper, let us call these particles as “fine particles”. On the other hand, for particles whose radii are in the range of \( r_{\text{cri}} < r < r_{\text{max}} \), the gravity force should be taken into account, since the rotation of crystal particles due to the magnetization force must finish before particles sediment to the bottom of vessel. In this paper, let us call these particles as “large particles”.

4.1 Rotation of fine particle with feeble magnetization and non-electric conductivity

An equation for rotational motion of a particle caused by magnetic field has been derived by Kimura,\(^{17}\) as shown in eq. (13)

\[
8\pi\eta^2 \frac{d\theta}{dt} + \frac{2\pi}{3\mu_0} r^3 \Delta\chi B^2 \sin 2\theta = 0
\]  
(13)

The first and the second terms in eq. (13) denote rotational torques caused by the liquid viscosity due to viscous resistance and by magnetic field, respectively. Here, \( \Delta\chi = \chi_1 - \chi_2 \) and \( \chi_1 \) is the magnetic susceptibility of a crystal axis having larger magnetization force, \( \chi_2 \) the magnetic susceptibility of another crystal axis having smaller magnetization force, \( \theta \) the angle between the crystal axis having larger magnetization force and the direction of imposed magnetic field.

The solution of eq. (13) is given by eq. (14).

\[
\tan \theta = \tan \theta_0 \exp(-t/\tau_m), \quad \tau_m = \frac{6\eta\mu_0}{\Delta\chi B^2},
\]  
(14)

where \( \theta_0 \) is initial value of \( \theta \) at \( t = 0 \), \( \tau_m \) is a time constant.

4.2 Rotation of fine particle with feeble magnetization and well electric conductivity

When such particle having well electric conductivity as metallic one is moved in a magnetic field, a rotational torque of a particle caused by Lorenz force is generated, as expressed by eq. (15)

\[
T_I = \frac{4}{15} \pi r^3 \sigma B^2 \frac{d\theta}{dt},
\]  
(15)

where \( \sigma \) is electric conductivity of particle. By taking the torque into consideration, an equation for the rotational motion of a particle caused by a magnetic field has been derived by Sugiyama et al.,\(^{16}\) as shown in eq. (16)
\[ 8 \pi \eta \frac{d \theta}{dt} + \frac{4}{15} \pi \sigma B^2 \frac{d \theta}{dt} + \frac{2 \pi}{3 \mu_0} r^3 \Delta \chi B^2 \sin 2 \theta = 0 \]  
(16)

The solution of eq. (16) is given as eq. (17), which shows the change in the angle of particle \( \theta \) with time \( t \), with the initial angle \( \theta_0 \) at \( t = 0 \).

\[ \tan \theta = \tan \theta_0 \exp(-t/\tau), \quad \tau = \frac{30 \eta + \pi \sigma B^2}{5 \Delta \chi B^2 - \mu_0} \]  
(17)

### 4.3 Rotation of large spherical particles

For large spherical particles, the gravity force does not affect their rotation due to magnetic field. Thus, Equations (13)–(17) derived above for the rotation of fine particle also hold for the rotation of the large particle. However, the rotation of the particles must finish before the completion of sedimentation of particles due to the gravity force.

### 5. Rotation of Rod-Shape Particle

In a similar manner to the case of spherical particle, the rotation of rod-shape particle can be theoretically analyzed to obtain the equations. It is also valid for the fine rod-shape particles that the Brownian motion will not affect the crystal alignment, although it overcomes the gravity motion. On the other hand, for the large rod-shape particle, the sedimentation of particles in a vessel due to the gravity force should be taken into account to analyze the rotation of the particles due to the magnetization force.

#### 5.1 Rotation of fine rod shape particle with feebly magnetization and non-electric conductivity

When a particle has rod shape, the torque of rotation due to magnetic field \( T_m \) is expressed as eq. (18)

\[ T_m = \frac{\pi}{8 \mu_0} d^2 l \Delta \chi B^2 \sin 2 \theta, \]  
(18)

where \( d \) is diameter of rod and \( l \) length.

When the rod particle rotates in a liquid, the liquid viscosity induces a torque \( T_d \) to resist against its rotation, as shown in eq. (19).\(^{24}\)

\[ T_d = \frac{1}{3} \eta l \frac{d \theta}{dt}, \]  
(19)

By considering the torques described above, the equation for the rotational motion caused by magnetic field is given by eq. (20)

\[ T_m + T_d = 0 \]  
(20)

Substituting eqs. (18) and (19) into eq. (20) yields eq. (21).

\[ \frac{1}{3} \eta l \frac{d \theta}{dt} + \frac{\pi}{8 \mu_0} d^2 l \Delta \chi B^2 \sin 2 \theta = 0 \]  
(21)

The solution of eq. (21) can be given as eq. (22)

\[ \tan \theta = \tan \theta_0 \exp(-t/\tau), \quad \tau = \frac{4 \mu_0 l^2 \eta}{3 \pi d^2 \Delta \chi B^2} \]  
(22)

#### 5.2 Rotation of fine rod shape particle with feebly magnetization and well electric conductivity

In a similar manner to the section 4.2, for fine rod shape particles having well electric conductivity, the torque of rotation caused by the Lorentz force should be taken into account to analyze the rotation.

At the position \((i, \theta, 0)\) in a rod particle with a length of \( l \), the rotation radius \( a \) and the rotation velocity \( v \) are expressed as follows:

\[ a = i \sin \theta \hat{x} + i \cos \theta \hat{z}, \]  
(23)

\[ v = i \cos \theta \frac{\partial}{\partial t} i \hat{x} - i \sin \theta \frac{\partial}{\partial t} i \hat{z}. \]  
(24)

When the electrically conductive particle rotates in a magnetic field, electric current is induced due to interaction of the rotational motion of particle and a magnetic field, as shown in eq. (25).

\[ j = \sigma v \times B = \sigma i B \cos \theta \frac{\partial}{\partial t} i \hat{x}. \]  
(25)

Here, the current path depends on the electric conductivities of liquid medium and suspended particles. When the electric conductivity is larger in the liquid medium than in the rod particles, the induced electric current is considered to flow in the medium. On the other hand, in the case where the medium is not electrically conductive, which corresponds to a case where metallic particles are suspended in a non-conductive medium, current loop will be closed inside of the particles. Thus, in this study, it is assumed that the liquid medium has larger electric conductivity than the particles. This assumption gives out the maximum value of Lorentz force due to the motion of the rod particle under magnetic field.

The Lorenz force as an electromagnetic force is generated by the interaction of the induced current and the imposed magnetic field. The Lorenz force \( f \) that acts on the particle to suppress its rotation and the torque \( T_e \) caused by the Lorenz force are derived as eqs. (26) and (27), respectively.

\[ f = j \times B = -\sigma i B^2 \cos \theta \frac{\partial}{\partial t} i \hat{x}, \]  
(26)

\[ T_e = a \times f = \sigma i B^2 \cos^2 \theta \frac{\partial}{\partial t} i \hat{x}. \]  
(27)

By integrating eq. (27) over the rod particle with a length of \( l \), the torque \( T_I \) that acts on the whole rod particle can be obtained as eq. (28)

\[ T_I = \frac{\pi d^2}{4} \int_{l/2}^{l/2} i^2 \sin \theta \cos \theta \frac{\partial}{\partial t} i \hat{x} = \frac{\pi \sigma d^2 B^2 l^3}{48} \cos^2 \theta \frac{\partial}{\partial t} i \hat{x} \]  
(28)

The equation for the rotational motion of fine rod shape particle caused by a magnetic field is given as eq. (29).

\[ \left( \frac{\pi \sigma d^2 B^2 l^3}{48} \cos^2 \theta + \frac{\eta \theta}{3} \right) \frac{\partial}{\partial t} \frac{\partial}{\partial t} + \frac{\pi \mu_0}{8 \mu_0} d^2 l \Delta \chi B^2 \sin 2 \theta = 0 \]  
(29)

If we assume \( \cos \theta = 1 \) in the first term of eq. (29), the nonlinear equation of eq. (29) can be simplified to a linear one of eq. (30).
(πσd²B²l³/48 + ηl³/3) dθ/dt + π/(8μ₀) d²lΔχB² sin 2θ = 0 \quad (30)

This assumption results in the maximum time constant \( τ_1 \). The solution of eq. (30) is given as eq. (31).

\[
\tan \theta = \tan \theta_0 \exp(-t/τ_1), \quad τ_1 = \frac{\pi \sigma d^2 B^2 + 16 \eta l^2 \mu_0}{12\pi d^2 \Delta \chi B^2} \quad (31)
\]

On the other hand, if we assume \( \cos \theta = 0 \), the minimum time constant \( τ_2 \) can be obtained as shown in eq. (32).

\[
τ_2 = \frac{4\eta l^2 \mu_0}{3π d^2 \Delta \chi B^2} \quad (32)
\]

The solution of time constant for eq. (29) should exist between \( τ_1 \) and \( τ_2 \).

### 5.3 Rotation of large rod particles

Let us consider the settling of a rod particle in a liquid medium due to the gravity force. The terminal settling velocity \( v \) of the rod particle is expressed by eq. (33).\(^{25}\)

\[
v = C_Dg(ρ_p - ρ)D_p^2/18\eta,
\]

where \( C_D \) is the drag coefficient of rod particle. The falling time \( t_f \) of the particle can be expressed by eq. (34)

\[
t_f = \frac{18Lη}{C_Dg(ρ_g - ρ)D_p^2} \quad (34)
\]

When the particle settles in a liquid medium, the gravity force will act on the particle, as shown in Fig. 3.

The torque caused on the rod particle by the gravity force is given by eq. (35)

\[
T_g = \frac{mg}{2}(d \cos \theta - l \sin \theta) \quad (35)
\]

Considering the relation between the direction of the gravity force and the magnetic field, two cases should be considered; one is the gravity force parallel to the direction of magnetic field and the other is perpendicular to the direction of magnetic field. In this paper, the magnetic field imposed in parallel to the direction of gravity force is called as “Pa magnetic field”. On the other hand, the magnetic field imposed in perpendicular to the direction of gravity force is called as “Pe magnetic field”.

When a Pa magnetic field is imposed, as schematically shown in Fig. 3, the magnetic field and the gravity force act on the rod particle from the opposite direction of each other. Thus, Equations (21) and (29) become eqs. (36) and (37), respectively.

\[
\frac{1}{3} \eta l^3 \frac{dθ}{dt} + \frac{π}{8μ₀} d^2lΔ\chi B^2 \sin 2θ + \frac{mg}{2}(d \cos θ - l \sin θ) = 0 \quad (36)
\]

\[
\left(\frac{πσd^2B^2l^3}{48} - \frac{ηl^3}{3}\right) \frac{dθ}{dt} + \frac{π}{8μ₀} d^2lΔ\chi B^2 \sin 2θ + \frac{mg}{2}(d \cos θ - l \sin θ) = 0 \quad (37)
\]

Let us consider the possible rotation direction of a rod particle with \( χ_e > χ_{k,b} \). In its long axis. When \( θ = π/2 \), the rod particle is considered to lie down on the bottom of a crucible. Therefore, particles are stable at that position.

In eqs. (36) and (37), the last two terms are same. Let us define \( E = \frac{Δ\chi B^2}{μ₀πd^2} \sin 2θ + \cos θ - \frac{1}{2} \sin θ \). Thus, if \( E > 0 \), that is \( \frac{dθ}{dt} < 0 \), then the rod particle will rotate along the magnetic field direction. On the contrary, if \( E < 0 \), namely, \( \frac{dθ}{dt} > 0 \), then the rod particle will orient perpendicular to the magnetic field direction. Here, let us define non-dimensional numbers \( C \equiv \frac{Δ\chi B^2}{μ₀πd^2} \) and \( A = l/d \) (aspect ratio). Then \( E = C \sin 2θ + \cos θ - A \sin θ \).

Figure 4 plots the curves of \( E \) versus \( θ \). According to the curve I, which corresponds to the case of Pa magnetic field, the value of \( E \) changes from positive to negative with increasing \( θ \). In other words, the rod particles can orient to the direction in parallel (for positive value of \( E \)) or in perpendicular (for negative value of \( E \)) to the magnetic field direction. That is, the orientation of the rod particle is also affected by the initial angle of the rod particle. Let us define a critical angle \( θ_0 \), where \( E(θ_0) = 0 \). The value of \( θ_0 \) is determined by the value of \( C \) and \( A \). For the case of \( C \gg A \), \( θ_0 \) approaches to \( π/2 \), while for \( C \ll A \), \( θ_0 \) approaches to 0. When \( θ < θ_0 \), the rod particles will rotate to the direction parallel to the magnetic field, while for the case of \( θ > θ_0 \), it rotates to the direction perpendicular to the magnetic field. Thus, in order to align the particles in parallel to the magnetic field, \( θ_0 \) should be large. Therefore, \( C \) should be large and \( A \) be small. That is, higher magnetic field and smaller particle length \( l \) are better for the alignment of the rod particles.

On the other hand, when Pe magnetic field is imposed, the magnetic field and the gravity force for orientation will reinforce each other. However the effect of the gravity force largely depends on the position of a rod particle in three-
dimension space. As shown in Fig. 5, a line l represents a rod particle and line m shows the component of the line l on the x–c plane, which is in parallel to the direction of the magnetic field. The l–m plane is the plane where the rotation takes place when the gravity force is not considered. By taking account of the action of gravity force on the rotation of particle due to the magnetic field, then \( E \) accounts of the action of gravity force on the rotation of field. The expression of gravity force can also be analyzed in a similar manner. When a magnetic field is imposed, both the magnetic field and the gravity force will promote the orientation of particle.

Moreover, for crystals whose magnetic susceptibilities in each crystal axes are expressed as \( \chi_c > \chi_{ab} \), small value of \( A \) is desired for Pa magnetic field and on the other hand, for crystals with \( \chi_c < \chi_{ab} \), large value of \( A \) is favorable for Pe magnetic field and small value for Pe magnetic field.

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### REFERENCES


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<td>a</td>
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<tr>
<td>U</td>
<td>Magnetization energy J/m²</td>
</tr>
<tr>
<td>υ</td>
<td>Rotation velocity m/s</td>
</tr>
<tr>
<td>V</td>
<td>Volume of particle m³</td>
</tr>
<tr>
<td>VT</td>
<td>Terminal settling velocity m/s</td>
</tr>
<tr>
<td>x(θ₀)</td>
<td>Displacement of Brownian motion m</td>
</tr>
<tr>
<td>η</td>
<td>Viscosity coefficient of medium Pa·s</td>
</tr>
<tr>
<td>θ</td>
<td>Angle between easy magnetization axis and direction of imposed magnetic field —</td>
</tr>
<tr>
<td>θ₀</td>
<td>The critical angle satisfy $E(θ₀) = 0$ under Pa magnetic field —</td>
</tr>
<tr>
<td>θ₀'</td>
<td>The critical angle satisfy $E(θ₀') = 0$ under Pe magnetic field —</td>
</tr>
<tr>
<td>μ₀</td>
<td>Permeability in vacuum (4π × 10⁻⁷) [H/m]</td>
</tr>
<tr>
<td>ρ₁</td>
<td>Density of particle kg/m³</td>
</tr>
<tr>
<td>ρ₂</td>
<td>Density of medium kg/m³</td>
</tr>
<tr>
<td>σ</td>
<td>Friction coefficient —</td>
</tr>
<tr>
<td>σ</td>
<td>Electric conductivity S/m</td>
</tr>
<tr>
<td>τ₀</td>
<td>Relaxation time of Rotational Brownian motion s</td>
</tr>
<tr>
<td>τ₀m</td>
<td>Magnetic alignment time s</td>
</tr>
<tr>
<td>τ₀R</td>
<td>Relaxation time of Brownian motion s</td>
</tr>
<tr>
<td>χ</td>
<td>Magnetic susceptibility —</td>
</tr>
<tr>
<td>Δχ</td>
<td>Difference between magnetic susceptibilities in crystal axes —</td>
</tr>
</tbody>
</table>