Mixing Time in a Bath Agitated Simultaneously by Bottom Gas Injection and Side Liquid Injection

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Effective mixing of a cylindrical bath is of crucial for wastewater treatment. Measurements were carried out on the mixing time in a cylindrical bath agitated simultaneously by bottom gas injection and side liquid injection. Air was injected into the bath through a centered single-hole bottom nozzle. Water was injected into the bath through a single-hole nozzle attached to the side wall of the vessel, drained from a pipe installed at the opposite bottom corner, and injected again with a small pump. The mixing time due to simultaneous gas and water injection, \(T_{m3}\), was satisfactorily correlated by the following empirical equation.

\[
\frac{1}{T_{m3}} = \frac{1}{T_{m1}} + \frac{1}{T_{m2}}
\]

where \(T_{m1}\) and \(T_{m2}\) are the mixing time values for water injection alone and gas injection alone, respectively. This relationship means that mixing by gas injection and that of liquid injection proceed in parallel with each other.

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1. Introduction

Concerning the mixing time of batch-type metals refining processes agitated by gas injection, remarkable progress was achieved by pioneering work of Nakanishi et al.\(^1\) Since then numerous investigations on the mixing time for various types of reactors agitated by gas injection have been carried out.\(^2\)\(^-\)\(^9\)

Gas injection techniques find wide applications in other fields of engineering. The present authors have applied the gas injection method to snow melting\(^10\) and wastewater treatment.\(^11\) The melting rate of snow and the efficiency of wastewater treatment were highly enhanced in the presence of the swirl motion.\(^8\),\(^9\)

In this study, particular attention was paid to the effect of liquid injection on the mixing time in a cylindrical water bath subjected to centered bottom gas injection. Water was injected into the bath through a circular nozzle installed flush-mount to the side wall of the vessel, drained through a pipe attached to the opposite bottom corner, and injected again to keep the bath surface constant. The water therefore was circulated. The mixing time was found to be independent of the tracer and sensor positions. An empirical equation was proposed for the mixing time in a bath agitated simultaneously by gas and water injection as a function of the mixing times for gas injection alone and water injection alone.

This agitation system can be applied to a batch type wastewater treatment. As mentioned above, the authors reported previously that refractory organic wastewater contained in a batch type vessel can be effectively processed by bottom gas injection.\(^11\) The gas was a mixture of air and ozone. The efficiency of the treatment would be highly enhanced by simultaneous gas and liquid injection. This is the main reason why mixing time was chosen to quantitatively evaluate the intensity of mixing in the bath. If we use vessels of more than two, we can develop a continuous process for wastewater treatment. In this case, however, the residence time must be taken into consideration in place of mixing time.

2. Experimental Apparatus and Procedure

2.1 Experimental apparatus

Figures 1 and 2 show schematic diagrams of the experimental apparatus and test vessel, respectively. The mixing time was defined as a period for an instantaneous tracer concentration to settle within \(\pm 5\%\) deviation around the final constant tracer concentration in the bath (called 95\% criterion), as shown in Fig. 3.

Aqueous KCl solution with a concentration of 1 kmol/m\(^3\) was used as tracer. The volumetric concentration of the tracer

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![Fig. 1 Schematic diagram of experimental apparatus.](image-url)
The electrical conductivity of liquid in the bath was measured using a sensor shown in Fig. 4. The time constant of the present system composed of the electric conductivity meter and a pen recorder was 0.25 s. The mixing time measurements were repeated ten times at each measurement position.

2.2 Effects of sensor and tracer positions on mixing time

Full understanding of the effects of sensor and tracer positions on the mixing time has not been obtained. We therefore investigated at first the effects of sensor and tracer positions on the mixing time.

2.2.1 Vertical sensor position

The radial sensor position was fixed at the position A \((r_s = 9\,\text{cm})\) in Fig. 2 and the sensor was placed at a vertical position of \(H_s = 1, 5, 10, 15, 20, \text{ or } 25\,\text{cm}\), where \(H_s\) was measured from the bottom of the bath. Tracer was added onto the bath surface at the radial position D of \(r_t = 9\,\text{cm}\), as shown in Fig. 2. The air and water flow rates, \(Q_g\) and \(Q_w\), were \(120\,\text{cm}^3/\text{s}\) and \(150\,\text{cm}^3/\text{s}\), respectively.

2.2.2 Radial sensor and tracer positions

The radial tracer position was fixed on the bath surface at the position D and the sensor was placed in turn at the position A,B,C or D. Later, the sensor was placed at the position D and the radial tracer position was shifted to the position A,B or C to examine the symmetry of mixing time with respect to the line connecting the inlet and exit of the forced circulation pipe line. The other parameters were chosen as follows: \(H_s = 1\,\text{cm}\), \(Q_g = 120\,\text{cm}^3/\text{s}\) and \(Q_w = 150\,\text{cm}^3/\text{s}\).

2.3 Effect of water flow rate, \(Q_w\), on mixing time

The sensor was placed at \(H_s = 1\,\text{cm}\) and \(r_s = 9\,\text{cm}\) (Position A), and tracer was added onto the bath surface at the position D. The water flow rate, \(Q_w\), was set at 0, 50, 100, or \(150\,\text{cm}^3/\text{s}\) for a gas flow rate of \(Q_g = 120\,\text{cm}^3/\text{s}\).
2.5 Effect of bath depth, \(H_L\), on mixing time

According to previous investigations, 13–15) a bottom blown bubbling jet rotates around the vessel axis when the ratio of the bath depth, \(H_L\), to the bath diameter, \(D\), satisfies a certain condition. This is called the swirl motion of the bubbling jet. Two kinds of swirl motions are known. One is observed for \(0 < \frac{H_L}{D} < 1\) and closely related to so-called rotary sloshing 16) appearing in a cylindrical bath subjected to external forced oscillation. The swirl radius, \(r_c\), of the bubbling jet on the bath surface is relatively small and expressed by \(r_c = 0.37R\), where \(R\) is the vessel radius. 8,9) The other is induced by a hydrodynamic instability of a large scale ring vortex enclosing the bubbling jet in a bath with \(H_L/D > 2\). This instability is closely associated with the Coanda effect 17) acting on a confined jet near a wall. The bubbling jet approaches the side wall of the vessel and ascends along it while swirling. These two kinds of swirl motions are called, for convenience, the first and second kinds of swirl motions, respectively. 13,14)

In this study the bath depth, \(H_L\), was changed from 5 cm to 30 cm. The first kind of swirl motion appeared for \(H_L = 10\) cm and 15 cm, but the second kind did not occur under the present experimental conditions. The sensor and tracer positions are the same as described in Section 2.4.

2.6 Effect of water injection height, \(H_{ni}\), on mixing time

Measurements were made for \(Q_w = 100\ \text{cm}^3/\text{s}\), \(Q_g = 120\ \text{cm}^3/\text{s}\), \(H_L = 30\) cm, and three water injection heights of \(H_{ni} = 10, 20,\) and 30 cm. The same sensor and tracer positions as described in Section 2.4 were chosen here.

3. Experimental Results and Discussion

For the sake of simplicity, we denote mixing time values for water injection alone, air injection alone, and simultaneous air and water injection by \(T_{m1}\), \(T_{m2}\), and \(T_{m3}\), respectively.

3.1 Effects of sensor and tracer positions on mixing time

The measured values of the mixing time for different vertical sensor positions are listed in Table 1. They are also plotted in Figs. 5 through 7 for individual water flow rate. The scatter of data points at each measurement position was indicated by an error bar. A dashed line for each experimental run denotes a mean value averaged in the axial, i.e., vertical direction. The measured \(T_{m3}\) values were scattered around the

![Fig. 5 Relation between mixing time and sensor position for \(Q_w = 50\ \text{cm}^3/\text{s}\).]
dashed line within a deviation of ±25%, being much smaller than the scatter of data points at each measurement position. Accordingly it can be said that there exists no definite dependence of $T_m$ on $H_s$ under the present experimental conditions.

Figure 8 shows the measured values of the mixing time for different radial sensor and tracer positions. The mixing time was independent of the radial sensor position as well as the tracer position.

### 3.2 Effect of water flow rate on mixing time

Measured values of the mixing time for different water flow rates are shown in Fig. 9 and Table 2. The following relation was found among $T_{m1}$, $T_{m2}$ and $T_{m3}$.

$$1/T_{m3} = 1/T_{m2} + 1/T_{m2}$$  \hspace{1cm} (1)

The dashed line in Fig. 9 denotes the value calculated from eq. (1). A good agreement between the measured and calculated values can be seen. Equation (1) has the same functional relationship as that for the combined electrical resistance in the case that two electrical resistances are connected in parallel. The mixing time is regarded as a measure for the flow resistance. A long mixing time value is realized when the flow resistance is high. Therefore, eq. (1) means that the mixing by gas injection proceeds in parallel with the mixing by liquid injection. Taking pictures of the flow field in the bath was difficult because many nozzles were attached to the side wall of the vessel. According to eye inspection of the flow field, however, the flow pattern produced by bottom gas injection in the bath seems to be hardly affected by water injection from the side wall of the vessel when eq. (1) is valid.

Measured mixing time values for water injection alone decreased in inversely proportion to the water flow rate $Q_w$. 

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Fig. 6 Relation between mixing time and sensor position for $Q_w = 100\text{cm}^3/\text{s}$.

Fig. 7 Relation between mixing time and sensor position for $Q_w = 150\text{cm}^3/\text{s}$.

Fig. 8 Mixing time as a function of sensor and tracer positions for $Q_w = 150\text{cm}^3/\text{s}$ and $Q_g = 120\text{cm}^3/\text{s}$.

Fig. 9 Relation between mixing time and water flow rate for $Q_g = 0$ and $120\text{cm}^3/\text{s}$.
as shown in Fig. 10. We derived the following empirical equation considering the dependence of the bath depth, \( H_L \), on \( T_{m1} \) as well.

\[
T_{m1} = 430H_L^{0.5}Q_w^{-1}
\]

\( 5 \text{ cm} \leq H_L \leq 30 \text{ cm} \),
\( 50 \text{ cm}^3/\text{s} \leq Q_w \leq 150 \text{ cm}^3/\text{s} \)  (2)

The adequacy of the relation between \( H_L \) and \( T_{m1} \) will be demonstrated in a later section.

Torii and Yang\(^{18}\) carried out model experiments on the mixing characteristics of gas-powder stirred ladle systems with through flow to determine optimum conditions for solid-liquid mixing. Gas was injected together with solid particles into a vessel with a rectangular cross section from a nozzle attached to one of four side walls of the vessel at a constant flow rate. Water was injected and subsequently ejected in a similar manner as described in this study but it was not circulated. The mixing time was defined as a period required for achievement of a uniform particle concentration in the bath. They determined the mixing time using an image processing technique and concluded that the contribution of liquid flow to the mixing time is very small. This result is appreciably different from the present experimental results.

Such a difference might be caused by the difference between the experimental facilities used by Torii and Yang\(^{18}\) and the present authors.

### 3.3 Effect of gas flow rate on mixing time

The relationship between the mixing time and air flow rate \( Q_g \) is shown in Fig. 11 and Table 3. The measured \( T_{m3} \) values decreased with an increase in \( Q_g \). The dependence of \( T_{m3} \) on \( Q_g \) is relatively weak compared with its dependence on the water flow rate \( Q_w \). Equation (1) can approximate the measured \( T_{m3} \) values satisfactorily.

![Fig. 10 Comparison of the measured values of mixing time with eq. (2).](image)

![Fig. 11 Relation between mixing time and air flow rate for \( Q_w = 0 \) and 100 cm\(^3\)/s.](image)

### Table 2 Mixing time values for different water flow rates.

<table>
<thead>
<tr>
<th>( Q_w ) (cm(^3)/s)</th>
<th>( Q_g ) mean</th>
<th>( Q_g ) max</th>
<th>( Q_g ) min</th>
<th>( Q_g ) mean</th>
<th>( Q_g ) max</th>
<th>( Q_g ) min</th>
<th>Estimation</th>
</tr>
</thead>
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<td>0</td>
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<td>—</td>
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<td>21.2</td>
<td>24.9</td>
<td>19.0</td>
<td>—</td>
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<td>12.0</td>
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<td>150</td>
<td>14.6</td>
<td>15.2</td>
<td>13.5</td>
<td>9.1</td>
<td>12.0</td>
<td>6.8</td>
<td>8.7</td>
</tr>
</tbody>
</table>

(= unit: s)

### Table 3 Mixing time values for different air flow rates.

<table>
<thead>
<tr>
<th>( Q_g ) (cm(^3)/s)</th>
<th>( Q_w ) mean</th>
<th>( Q_w ) max</th>
<th>( Q_w ) mix</th>
<th>( Q_w ) mean</th>
<th>( Q_w ) max</th>
<th>( Q_w ) mix</th>
<th>Estimation</th>
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<td>9.3</td>
<td>11.2</td>
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<td>14.4</td>
<td>15.0</td>
<td>13.8</td>
<td>9.9</td>
<td>12.4</td>
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<td>9.5</td>
<td>7.9</td>
<td>8.5</td>
</tr>
</tbody>
</table>

(= unit: s)
Previous investigators\(^1\textsuperscript{–}^5\) found that the mixing time, \(T_{m2}\), in a bath stirred by bottom gas injection alone can be correlated as a function of the specific mixing power, \(\varepsilon\), defined by

\[
\varepsilon = \frac{\rho_g g Q_g H_L}{(\pi \rho_c H_L D^2)/4} = 4g Q_g/(\pi D^2)
\]

(Nakanishi et al.\(^1\)) investigated systematically the mixing time in a bath stirred by bottom gas injection and reported that \(T_{m2}\) is proportional to \(\varepsilon^{0.4}\). Nowadays it is generally accepted that the power of \(\varepsilon\) is dependent on the mode of energy input and the vessel geometry.\(^5,12,19\textsuperscript{–}22\)

The same functional relationship as derived by Nakanishi et al.\(^1\) was valid for the presently measured values, as shown in Fig. 12. The solid line denotes an empirical correlation of \(T_{m2}\) expressed by

\[
T_{m2} = 190\varepsilon^{-0.4}
\]

\(10 \text{ cm} \leq H_L \leq 30 \text{ cm}, \quad 10^3 \text{ cm}^2/\text{s}^3 < \varepsilon < 10^3 \text{ cm}^2/\text{s}^3\)

(Nakanishi et al.\(^1\)) gave a coefficient of 240.

Tortii and Yang\(^18\)) derived two empirical correlations having different dependence on \(\varepsilon\), just like Asai et al.\(^21,22\).

### 3.4 Effect of bath depth on mixing time

Measured values of the mixing time for different bath depths are shown in Fig. 13 and Table 4. In the case of water injection alone, \(T_{m1}\) increased in proportion to \(H_L^{0.5}\). On the other hand, for gas injection alone, \(T_{m2}\) remained almost unchanged for \(H_L \geq 10 \text{ cm}\) but increased significantly as \(H_L\) became smaller than 10 cm. This is because injected gas leaves out of the bath without transferring most of its energy to water when the bath is shallow. Such a phenomenon is referred to as “blow out”. The mixing time, \(T_{m3}\), also increased as \(H_L\) became smaller than 10 cm. According to a previous study,\(^23,24\)) a bath contained in a cylindrical vessel can be classified into two types: shallow and deep baths. A motion of a shallow bath is influenced by the bottom wall of the vessel in addition to the side wall of the vessel. On the other hand, a motion of a deep bath is influenced only by the side wall. Therefore, a liquid in a deep bath is much more easier to move than that in a shallow bath. In other words, agitation of a shallow bath is not easy compared to a deep bath. The aspect ratio, \(H_L/D\), for the boundary between the two types of baths is 0.3. This critical aspect ratio is 6 cm in the present experiment as the vessel diameter, \(D\), is 20 cm. This may be another reason why the mixing time increases for \(H_L < 10 \text{ cm}\).

The first kind of swirl motion took place for \(H_L = 10 \text{ cm}\) and 15 cm when the bath was agitated by gas injection alone. The same kind of swirl motion occurred in the bath accompanied by simultaneous gas and water injection but it did not affect the mixing time just like the case of gas injection alone.\(^8,9\)) Figure 13 indicates that eq. (1) can

\[T_{m2} = 190\varepsilon^{-0.4}\]

\(10 \text{ cm} \leq H_L \leq 30 \text{ cm}, \quad 10^3 \text{ cm}^2/\text{s}^3 < \varepsilon < 10^3 \text{ cm}^2/\text{s}^3\)

\[
Q_g = 120 \text{ cm}^3/\text{s}, \quad Q_g = 120 \text{ cm}^3/\text{s}
\]

(Estimated)

\[\text{Bath height } H_L \text{ (cm)}\]

\[\text{Mixing time } T_m \text{ (s)}\]

### Table 4 Mixing time values for different bath depths.

<table>
<thead>
<tr>
<th>(H_L) (cm)</th>
<th>(Q_w)</th>
<th>(Q_g)</th>
<th>(Q_w)</th>
<th>(Q_g)</th>
<th>(Q_w)</th>
<th>(Q_g)</th>
<th>(Q_w)</th>
<th>(Q_g)</th>
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<td>11.2</td>
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</tbody>
</table>

(unit: s)
reasonably approximate the measured $T_{m3}$ values in the absence of the blow out phenomenon for $H_L > 10$ cm.

3.5 Effect of injection height of water, $H_{ni}$, on mixing time

Experimental results of the mixing time are shown against the nozzle height, $H_{ni}$, in Fig. 14 and Table 5. The measured values are in good agreement with eq. (1) for $H_{ni}$ from 10 cm to 30 cm.

3.6 Effects of vessel diameter, $D$, ejection height of water, $H_{ne}$, and other parameters on mixing time

Effects of influential parameters such as $D$ and $H_{ni}$ on the mixing time, $T_{m3}$, will be investigated in a future study.

4. Conclusions

A fundamental investigation was carried out on the mixing time in a cylindrical bath agitated simultaneously by bottom gas injection and side liquid injection. For example, information on the mixing time is useful for developing a new batch type process for wastewater treatment. The results obtained in this study are summarized as follows:

(1) The mixing time in a bath with simultaneous air and water injection, $T_{m3}$, is nearly constant in the whole bath irrespective of the sensor and tracer positions.

(2) In the absence of the blow out phenomenon, the mixing time in a bath with simultaneous gas and water injection, $T_{m3}$, can be correlated by the following empirical equation.

\[
\frac{1}{T_{m3}} = \frac{1}{T_{m1}} + \frac{1}{T_{m2}}
\]

(1)

where $T_{m1}$ and $T_{m2}$ denote the mixing time values for water injection alone and air injection alone, respectively. This equation has the same functional relationship as that for the combined electrical resistance in the case that two electrical resistances are connected in parallel. Therefore, mixing by gas injection and that by water injection proceed in parallel with each other.

(3) The measured values of $T_{m1}$ and $T_{m2}$ are approximated by the following empirical equations under the present experimental conditions.

\[
T_{m1} = 430H_L^{0.5}Q_{w}^{-1}
\]

(2)

\[
T_{m2} = 190\varepsilon^{-0.4}
\]

(4)

where $H_L$ is the bath depth, $Q_w$ is the water flow rate, and $\varepsilon$ is the specific mixing power.

Nomenclature

$D$: inner diameter of vessel (cm)

$d_n$: inner diameter of bottom nozzle for gas injection (cm)

$d_{nw}$: inner diameter of side nozzle for water injection (cm)

$H_L$: bath depth (cm)

$H_{ne}$: nozzle height for water drainage (cm)

$H_{ni}$: nozzle height for water injection (cm)

$H_s$: height of electric conductivity sensor (cm)

$Q_g$: gas flow rate (cm$^3$/s)

$Q_w$: water flow rate (cm$^3$/s)

$r_s$: radial sensor position (cm)

$r_t$: radial tracer addition position (cm)

$T_{m1}$: mixing time (s)

$T_{m1}$: mixing time for water injection alone (s)

$T_{m2}$: mixing time for air injection alone (s)

$T_{m3}$: mixing time for simultaneous water and gas injection (s)

$\varepsilon$: specific mixing power (cm$^2$/s$^3$)

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