Prediction of the Forming Limit of Porous Metals Using the Finite Element Method

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To predict the forming limit diagram of porous metals, we use the elasto-plastic finite element method in conjunction with a critical relative density criterion to simulate the deformation behavior in the upsetting of porous metallic specimens. We predict the strain instability of porous cylindrical specimens by calculating the strain paths during deformation. Strain instability is the point where axial strain starts to increase in compression. By using strain instability as a ductile fracture criterion of porous metals, we calculate a forming limit that agrees well with the experimental forming limit. The calculated relative density in crack initiation is higher than the critical relative density.

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1. Introduction

Powder metallurgy, in its earliest form, actually involved powder forging and the modern powder metallurgy industry was built on its capabilities for economical production of net shape parts, where dimensional precision outweighs the importance of mechanical properties.\textsuperscript{1} Powder forming involves fabrication of a preform by conventional press-and-sinter processing, followed by various forming processes,\textsuperscript{2} for example pulse electrical sintering,\textsuperscript{3} dynamic compaction,\textsuperscript{4,5} hot pressing,\textsuperscript{5,6} powder forging, powder extrusion, powder rolling \textit{etc.}, in which the powder or porous preform is brought into a final shape through substantial densification. To achieve a successful forming process without cracks, it is useful to know the forming limit of the material under investigation. During the forming process, cracks generally form as a result of a ductile fracture mechanism.\textsuperscript{7} A ductile fracture is a complex phenomenon that occurs as a result of the relation between stress, strain and material microstructure. The material damage is therefore affected not only by the material properties but also by the tool and specimen geometries, the friction conditions and the loading conditions. Researchers have proposed many theories to explain the ductile fracture. For example, the pore growth model, the local thinning model, the critical deformation energy model, the upper bound theory, and the tensile stress criterion.\textsuperscript{8–11}

One useful technique for evaluating the fracture under the conditions of a deformation process is the upset test.\textsuperscript{10} During axial compression of a cylinder, the friction at the die contact surface generally retards the radial outward flow of material at these surfaces and leads to barreling of the cylinder’s free surface. To obtain a forming limit diagram (FLD) of a material for process design applications, we need to perform many experiments under various specimen geometries and friction conditions.

Void formation at inclusions or other structural inhomogeneities initiates ductile fractures during plastic deformation of a fully dense material. The preexistence of numerous initial voids in porous materials eliminates the need for the first step in the ductile fracture process; only the coalescence of voids is necessary. Because the ductile fracture of metals occurs as a result of the generation and growth of voids, we can analyze the ductile fracture phenomenon with respect to the plastic behavior of porous metals. Indeed, Gurson’s yield function, which is used for the plasticity analysis of porous materials, has been widely used for fracture analysis.\textsuperscript{11}

We propose a new method (fracture criterion) for predicting the forming limit of porous metals, by using the finite element method and adopting the proposed fracture criteria.

2. Pressure-Dependent Constitutive Model and Calculation

Considering that yielding occurs when the strain energy of the porous metals including shear deformation energy and volumetric strain energy reaches a critical value, we obtained the following expression for yield function $F$,\textsuperscript{12}

\begin{equation}
F = AJ_1^2 + BJ_2^2 - \eta Y_0^2,
\end{equation}

where, $J_1$ is the first stress invariant, $J_2'$ is the second deviatoric stress invariant, and $\eta = Y_0^2 / Y_0^2$ ($Y_0$ is the apparent yield stress of the porous material and $Y_0$ is the yield stress of the matrix material), $A$ and $B$ are functions of the relative density $R$. For non-porous metals, $A = 3$, $B = 0$, $\eta = 1$ and the function $F$ is the von Mises yield function. Here $\eta$ is a geometric hardening parameter that shows the density dependency of the yield stress: $0 \leq \eta \leq 1$ as $R_c \leq R \leq 1$. Here $R_c$ is the adjustable experimental parameter in which the yield stress of the material becomes zero. The present authors proposed the following pressure-dependent yield function for porous materials,\textsuperscript{12}

\begin{equation}
F = (2 + R^2)J_1^2 + \frac{1 - R^2}{3} - \left(\frac{R - R_c}{1 - R_c}\right)^2 Y_0^2.
\end{equation}

From eq. (2), it is clear that the point when $R = R_c$ is the limit that the material can be loaded before fracture. In actual case, fracture occurs before the material totally loses its strength. By generating various strain paths, fracture points can be obtained and the forming limit line can be obtained by linking these points.
The elasto-plastic finite element analysis of porous metals has been done using the above yield function. The yield function $F$ is used to calculate the plastic strain increments as follows.

$$
\delta e_{ij}^p = \lambda \frac{\partial F}{\partial \sigma_{ij}} = \frac{q^T C^E \delta e}{p^T q + q^T C^E q \delta \sigma_{ij}},
$$

$$
d\sigma = C^E (\delta e - \delta e^p) = C^{EP} \delta e
$$

where $\lambda$ is a positive scalar, $C^E$ is an elastic stress-strain matrix, $C^{EP}$ is the relative density dependent elastoplastic stress-strain matrix, and $p^T$ and $q^T$ are transposes of $p$ and $q$, which are defined by eqs. (5)–(7).

$$
p = [p_{11} p_{22} p_{33}] [p_{12} p_{23} p_{13}],
$$

$$
q = [q_{11} q_{22} q_{33}] [q_{12} q_{23} q_{13}],
$$

$$
p_{ij} = -\partial F/\partial \sigma_{ij} \quad \text{and} \quad q_{ij} = \partial F/\partial \sigma_{ij}.
$$

The elastic stress-strain matrix $C^E$ is given by

$$
C^E = \frac{E(1-v)}{(1+v)(1-2v)} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Equations (3)–(8) are also valid in the case of non-porous materials.\(^{10}\) The relationship between the apparent plastic energy increment per unit volume of porous metal $dW$ and that of the non-porous metal $dW_0$ is given by

$$
dW = RdW_0 \quad \text{or} \quad \sigma ds_{ij}^p = RY_0 d\varepsilon_{ij},
$$

where $d\varepsilon_{ij}$ is the effective plastic strain increment of nonporous base metal. Since the yield stress is a function of the plastic work per unit volume, we can evaluate $p_{ij}$ and $q_{ij}$ using the following equations,

$$
p_{ij} = \frac{2\eta}{3R} \left[ \frac{EE_T}{E-E_T} \right] \sigma_{ij},
$$

$$
q_{ij} = \frac{2 + R^2}{3} \sigma_{ij} + \frac{2}{9} (1 - R^2) J_1 \delta_{ij},
$$

where $E$, $E_T$, $\sigma_{ij}$ and $\delta_{ij}$ are the elastic modulus and tangential modulus of non-porous base metal, deviatoric stress and the Kronecker delta respectively. For non-porous metals, $R = 1$ and $\eta = 1$, and eqs. (10) and (11) become those of the Von Mises yield condition. The elastoplastic stress-strain matrix $C^{EP}$ and the positive scalar $\lambda$ can be formulated using the eqs. (3), (4), (8), (10) and (11).

Once strains are known, the evolution of the density can be calculated using the following mass conservation equation during the deformation of the porous material:

$$
R_0 l_1 l_2 l_3 = R(l_1 + \Delta l_1) l_2 (l_2 + \Delta l_2) l_3 (l_3 + \Delta l_3),
$$

where $R_0$ and $R$ are the relative densities before and after deformation, and $l_1$, $l_2$ and $l_3$ are linear dimensions of a material element along the $x_i$ axis before and after deformation. It follows from eq. (12) that

$$
R = R_0 e^{-\epsilon_1 - \epsilon_2 + \epsilon_3} = R_0 e^{-\epsilon},
$$

where $\epsilon$ is the volumetric strain.

Kuhn performed the upset test on sintered Al alloy specimens and obtained the locus of the surface strain at the fracture.\(^{13}\) To compare our FLDS with Kuhn’s experimental FLDS, we analyzed the upsetting process of the sintered Al alloy. To generate various strain paths, we changed the geometry of the specimen to cylinders with aspect ratios (height/diameter) of 2, 1 and 0.5, thereby producing a flanged shape and a slanted shape. Figure 1 shows the initial mesh and boundary conditions. Although we can get various strain paths by varying the friction between the die and the specimen, we obtained a wide range of strain paths to fit the connecting line just by varying the specimen geometry. As a result, we set the friction coefficient $\mu$ to a constant value of 0.2. To obtain a homogeneous strain path, we also calculated a frictionless case.

The material we considered was a sintered 601AB Al alloy with an initial relative density of 0.788.\(^{13}\) The mechanical properties are as follows: Young’s modulus, 69.7 GPa; Poisson’s ratio, 0.3; yield stress, 68 MPa; flow curve, $\sigma = 206.85e^{0.25}$ (MPa).\(^{14,15}\) Figure 2 shows the variation of the
yield stress of the sintered 601AB with its density; the best fitting of it gives rise to the critical relative density $R_c = 0.367$. We performed the finite element simulation until the lowest density reached $R_c$.

3. Results and Discussion

Figure 3 shows the deformed meshes of specimens with various geometries when the density of the surface element with the lowest value reaches the critical relative density $R_c$. Represented by the dashed elements in Fig. 3, the surface of the equatorial zone is the farthest surface from the die and has the lowest density. As a result, the fracture is expected to occur at this region.

Figure 4 shows the calculated axial strain $\varepsilon_{zz}$, the hoop strain $\varepsilon_\theta$, the relative density $R$ and the hydrostatic pressure $\sigma_m$ at the dashed surface elements as a function of the punch stroke. Figure 4(a) shows the homogeneous deformation at an aspect ratio of 2. Because of the frictionless condition, $\varepsilon_\theta$ increases monotonously, $\varepsilon_{zz}$ decreases monotonously, the
density increases and the hydrostatic pressure decreases. Figures 4(b), (c) and (d) are, respectively, for the aspect ratios of 0.5, 1 and 2 at $\mu = 0.2$. When $\mu$ is greater than zero, $\varepsilon_{zz}$ and $R$ increase at first, then decrease beyond a certain point. The slanted and flange specimens are for aspect ratios between 1 and 0.5. In Figs. 4(a), (b) and (c), as the specimens are compressed, $\varepsilon_{zz}$ increases monotonously and $\varepsilon_{zz}$ decreases initially but increases when the axial tensile stress develops owing to bulging. The point at which the $\varepsilon_{zz}$ starts to increase coincides with the point at which the density starts to decrease. However, the density is not lowered to $R_c$. That is, the fracture occurs before the density reaches the critical value $R_c$. This increase of $\varepsilon_{zz}$ in the compression formability test of bulk materials is reported as strain instability and occurs in solid and in porous metals.\textsuperscript{13,16} This plateau is the point where the local $\varepsilon_{zz}$ change is zero, that is, a plane strain. Kuhn and Lee observed that the crack propagated rapidly at this point. They defined it as the starting point of the ductile fracture.\textsuperscript{16} We set this plateau as the fracture point. This strain instability occurs for all cylinders with aspect ratios of 2, 1, and 0.5. By connecting these instability points for each cylinder, we obtained the FLD.

Figure 5 shows the strain paths for the compression of the cylindrical specimens with different aspect ratios. The bottommost line is for the homogeneous compression. For a fully dense material, the gradient (hoop strain/axial strain, $\varepsilon_{00}/\varepsilon_{zz}$) is $-0.5$ due to the volume constancy during the plastic deformation. In contrast, the diameter of a sintered cylinder expands to a much less extent than a fully dense metal during the compressive deformation for a given reduction in height, and the volume decreases. Therefore, the curve is below the straight line of gradient $-0.5$; the axial strain decreases, the density increases and the gradient approaches $-0.5$.

By examining the effect of the aspect ratio of a cylinder, we observe that as the aspect ratio increases the instability strain increases and larger plastic deformation is possible. In Fig. 5, the instability strain is the point where the axial strain starts to increase. The strain instability points that we calculated fit well with the forming limit points that were found experimentally. For each curve, the last point of the strain path corresponds to $R = R_c$. Because the last point of the strain paths is beyond the forming limit, the relative density criterion cannot be used as a ductile fracture condition. We interpret this critical density point as merely the upper limit of crack formation and complete fracture occurrence. The FLD obtained by this plastic theory can be effectively used to design powder forming processes.

4. Conclusions

To predict the FLD of porous metals, we conducted finite element analysis based on a pressure-dependent constitutive model in the axisymmetric upsetting of a specimen. We predicted the strain instability by calculating the strain paths during their deformation. By using the onset of strain instability where the axial strain starts to increase in compression as a ductile fracture criterion, we calculated an FLD that agrees well with the experimental FLD. The calculated relative density in crack initiation is higher than the critical relative density $R_c$.

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REFERENCES